Workshop "Étale and motivic homotopy theory", Titles and Abstracts

Aravind Asok: *Rational points and zero cycles of degree* 1 *in* $\mathbb{A}^1$*-homotopy theory.*

I will discuss how sensitive different categories of homotopy theoretic origin are to existence of rational points or zero cycles of degree 1.

Thomas Geisser: *Rojtman's theorem for normal schemes*

We first discuss descent theorems for motivic homology theories (like Suslin homology) with respect to proper hypercoverings. This is used to extend Rojtman's theorem to normal schemes, relating the torsion in Suslin homology to the torsion of the Albanese scheme.

Olivier Haution: *Rost's degree formula in characteristic two*

The degree formula gives a relation between the Segre number of an algebraic variety (a geometric invariant) and its index (an arithmetic invariant). The formula can be used to prove incompressibility of varieties, and has applications to quadratic forms. I will discuss these classical results (mostly due to Rost), and give some new results over fields of characteristic two.

Armin Holschbach: *Étale contractible varieties in positive characteristic*

Homotopy theory is founded on the idea of contracting the interval, either as a space, or as an actual homotopy, i.e., a path in a space of maps. In algebraic geometry, the affine line $\mathbb{A}^1_k$ serves as an algebraic equivalent of the interval, at least in characteristic 0. Matters differ in characteristic $p > 0$ where $\pi_1(\mathbb{A}^1_k)$ is an infinite group. This raises the question whether there is an étale contractible variety in positive characteristic. In this talk, we show that there are no non-trivial smooth varieties over an algebraically closed field k of characteristic $p > 0$ that are contractible in the sense of étale homotopy theory. This talk is based on joint work with Johannes Schmidt and Jakob Stix.

Annette Huber–Klawitter: *Motives of commutative algebraic groups*

We report on joint work with Giuseppe Ancona and Stephen Enright-Ward. We establish that the motive of a commutative algebraic group scheme over a field has the expected shape. It is a symmetric algebra over its 1-motive. This 1-motive has an explicit description as homotopy invariant sheaf with transfers, it is nothing but the sheaf given by the group itself. In fact, we have a decomposition into Kuenneth components with the 1-motive in degree 1.

The proof is quite involved. One ingredient is a finiteness argument and a good understanding of the behaviour of the ℓ-adic realization. As a byproduct we establish Kimura finiteness of 1-motives and hence get a new proof of Kimura finiteness of motives of curves.

Moritz Kerz: *Deformation of algebraic cycle classes in characteristic zero*

We study formal deformation of algebraic cycle classes over a discrete valuation ring in characteristic zero. This is motivated by Grothendieck's variational Hodge conjecture, which in particular predicts which cycle classes should deform.

Marc Levine: *On the geomteric part of oriented theories*

(joint work with Girja Tripathi) There are two parallel theories of oriented cohomology for algebraic varieties: a purely algebraic theory, typified by the classical Chow ring or the Grothendieck group of vector bundles, and a motivic version, such as motivic cohomology or the full theory of algebraic K-theory. Each side has their own advantages and disadvantages. We will describe a series of comparison results that allow one to compute the geometric part of a motivic theory in purely algebraic terms, starting with (geometric) algebraic cobordism.

Niko Naumann: *Operations and E_∞-structures*

The first part of the talk will review operations in classical homotopy theory and try to advertise them by explaining the recent solution of a conjecture of J. P. May. The second part of the talk will address motivic variants of this.

Gereon Quick: *The Abel–Jacobi map and homotopy theory*

Calculating the residues for rational integrals in complex variables is a classical problem in mathematics. It is directly related to questions on algebraic cycles, their cohomology classes, and the Abel–Jacobi map. In this talk I will present joint work with Michael Hopkins in which we use topological cohomology theories to shed some new light on the Abel–Jacobi map.

Oliver Röndigs: *Some comments on the Grothendieck ring of varieties*

This talk will discuss some properties of the Grothendieck ring of varietes and related constructions from the perspective of motivic stable homotopy theory.

Andreas Rosenschon: *Etale motivic cohomology and algebraic cycles*

We consider the etale motivic or Lichtenbaum cohomology groups of a smooth projective variety and show that there are integral formulations of the Hodge and Tate conjecture for the Lichtenbaum Chow groups which are equivalent to the usual rational conjectures for the classical Chow groups.

Markus Spitzweck: *Triangulated categories of motives over general base schemes*

We discuss a construction of a motivic Eilenberg–MacLane spectrum over Dedekind domains and show that this gives rise to well behaved categories of motives over general schemes. In the second part we focus on mixed Tate motives, triangulated and abelian.

Shuji Saito: *Relative motivic complex with modulus and regulator maps*

The recent work of Kahn–Saito–Yamazaki on recirocity sheaves suggests a possibility of enlarging Voevodsky's triangulated category of motives to a larger category of motivic nature encompassing non-homotopy invariant phenomena. I will take some time for a speculation of (conjectural) reciprocity motives and then introduce a new non-homotopy invarint which is expected to be motivic cohomology with compact support in the context of reciprocity motives. For this I will define relative motivic complex as a complex of Zariski sheaves for a pair of a smooth variety and its effective (non-reduced) Cartier divisor. Its cohomology groups called relative motivic cohomology are related to various non-homotopy invariants such as relative Picard groups, relative Chow groups with moduli, and additive higher Chow groups by Bloch–Esnault–Park. The main results are computation of the motivic complex in weight one, and computation of relative motivic cohomology using relative Milnor K-groups, and the construction of regulator maps to a relative version of Deligne cohomology, which provides Abel–Jacobi maps with additive parts.

Rin Sugiyama: *Motivic homology of semiabelian varieties*

For an abelian variety, Beauville proved a vanishing of an eigenspace of Chow groups of an abelian varieties with respect to the pull-back along multiplication by integers. In this talk, we generalize this result for semiabelian varieties via motivic homology, using a recent result of Ancona–Enright-Ward–Huber on a decomposition of the motive of a semiabelian variety in the Voevodsky's category and a result of Kahn–Yamazaki on a description of some motivic homology in terms of K-groups attached to semiabelian varieties.

Kirsten Wickelgren: *A computational approach to the section conjecture*

Grothendieck's section conjecture predicts that rational points on hyperbolic curves X over number fields k are in bijection with conjugacy classes of sections of $\pi_1(X) \to \pi_1(k)$. Part of this conjecture reduces to $X = \mathbb{P}^1 - \{0, 1, \infty\}$. Conjugacy classes of sections are π_0 of a mapping space of étale homotopy types. We resolve the étale homotopy type of $\mathbb{P}^1 - \{0, 1, \infty\}$ to study this mapping space.

Jörg Wildeshaus: *Weights and conservativity*

Finite dimensionality à la Kimura provides a natural context to study conservativity of realizations of Chow motives over a point. The aim of the talk is to study conservativity phenomena in a context which will be more general in two respects: homological and geometrical. As far as the homological aspect of the question is concerned, the right generalization of finite dimensionality turns out to be primality à la André-Kahn. The geometrical context that will be studied is that of Beilinson motives over an arbitrary base; it is thanks to that choice that one has at one's disposal an important tool, namely, a weight structure à la Bondarko. Under appropriate hypotheses, the realization can then be shown to be conservative; in fact, it even turns out to be "weight-conservative" in a sense that will be made precise. Time permitting, an application of weight-conservativity to rigidification of certain motivic objects will be sketched.

Olivier Wittenberg: *On the cycle class map for zero-cycles over local fields*

The Chow group of zero-cycles of a smooth and projective variety defined over a field k is an invariant of an arithmetic and geometric nature which is well understood only when k is a finite field (by higher-dimensional class field theory). In this talk, we will discuss the case of local and strictly local fields. We prove in particular the injectivity of the cycle class map to integral ℓ-adic cohomology for a large class of surfaces with positive geometric genus over p-adic fields. The same statement holds for semistable $K3$ surfaces over $\mathbb{C}((t))$, but does not hold in general for surfaces over $\mathbb{C}((t))$ or over the maximal unramified extension of a p-adic field. This is a joint work with Hélène Esnault.

Marcus Zibrowius: *The γ-filtration on Grothendieck–Witt rings*

One of the three players in the *Milnor conjectures* for fields is the Witt ring and its filtration by powers of the fundamental ideal. In this talk, we exhibit a possible generalization of this filtration to Grothendieck–Witt rings of schemes: we study exterior power operations on these rings, show that the resulting (pre-)λ-structures are special, and thus obtain a so-called γ-filtration. The talk will close with a few example calculations and open questions.