Workshop "Étale and motivic homotopy theory", Titles and Abstracts

Piotr Achinger: *Log geometry, monodromy, and Betti realization of varieties defined by formal power series.*

A now classical construction due to Kato and Nakayama attaches a topological space ("Betti realization") to a log scheme over C. I will show how this construction can be used to recover the topological germ of a log smooth degeneration from the log special fiber alone. In particular, I will show how to compute the monodromy of the family directly from the log special fiber. In a different direction, I will also explain how Kato-Nakayama spaces can be employed to functorially attach homotopy types to varieties defined over the field $C((t))$ of formal Laurent series. For varieties defined by convergent series, this construction agrees with the usual one, obtained via complex topology. The first half is joint work with Arthur Ogus, the second is joint with Mattia Talpo. Almost no prior exposure to log geometry will be required.

Joseph Ayoub: *On the conservativity conjecture I & II*

The conservativity conjecture predicts that an algebraic correspondance between Chow motives is invertible if and only if its action on cohomology is invertible. I'll describe parts of work in progress aiming at proving this conjecture in characteristic zero and for a classical Weil cohomology theory.

Giulia Battiston: *Gauß Manin stratifications and a Künneth formula*

Extending the work of Phùng on Gauß Manin stratifications to the non relatively compact case, I will deduce a Künneth formula for the regular singular stratified fundamental group, which can be seen as a generalization of the tame fundamental group. As an application I will deduce that for homogeneous spaces the tame fundamental group is trivial if and only if the regular singular is, which is a generalization of a conjecture of Gieseker proposed by Esnault.

David Carchedi: *Galois Equivariant Étale Realization*

Let k be a base field. We will explain how the étale realization functor from motivic spaces over k to (a localization of) pro-spaces can be improved to a functor taking values in a certain localization of pro-objects in spaces with an action of the profinite Galois group $\mathrm{Gal}(k^{sep}/k)$, and explain how the original étale realization functor is essentially obtained by taking a homotopy quotient. We will then explain how to possibly extend this construction to a functor out of the stable motivic ∞-category over k to an interesting stable ∞-category associated to $\mathrm{Pro}(\mathrm{Gal}(k^{sep}/k)-\mathrm{Spc})$. Time permitting, we will list some applications. This is joint work with Elden Elmanto

Thomas Geisser: *Relating Tate's conjecture and the finiteness of the Tate–Shafarevich group*

It is a classical result of Artin–Grothendieck that given a smooth and proper surface X over a finite field which factors through a smooth and proper curve C, then the Brauer group of X is finite if and only if the Tate–Shafarevich group of the Jacobian of the generic fiber of X/C is finite. We will discuss a generalization of this result to X of arbitrary dimension.

Katharina Hübner: *Aspherical neighborhoods on arithmetic surfaces*

Consider an arithmetic surface X over some Dedekind scheme B and a prime number ℓ which is invertible on X. We investigate sufficient conditions for the ℓ-completion of the étale homotopy type $X_{ét}$ to be aspherical. In case B is the ring of integers of some local field this allows us to construct for every geometric point of a given arithmetic surface over B a basis of étale neighborhoods which are aspherical with respect to ℓ. In the global case we construct a basis of neighborhoods which has this property after suitable (ramified) extension of the base scheme B.

Moritz Kerz: *Algebraic K-theory and descent for blow-ups II*

This talk is a continuation of part I (see Florian Strunk).

Oliver Röndigs: *Computations of stable homotopy groups of motivic spheres over a field*

The talk will report on joint work (partly in progress) with Markus Spitzweck and Paul Arne Østvær. This work describes the 1-line and the 2-line of stable homotopy groups of the motivic sphere spectrum over a field of characteristic not two. More precisely, the kernel of the unit map to an appropriate connective cover of hermitian K-theory can be described in these lines via Milnor K-theory and motivic cohomology. The main computational tool is Voevodsky's slice spectral sequence.

Robert Kucharczyk: *Topological models for absolute Galois groups*

In this talk I will present two analogous constructions, one in the world of schemes over the complex numbers and one in the world of topological spaces, of a functorial étale $K(\pi, 1)$ scheme / space for a large class of absolute Galois groups. In particular we obtain such a space for the maximal abelian extension of the rationals. The classical fundamental group, defined in terms of paths, is then a remarkable dense subgroup of the absolute Galois group. This is joint work with Peter Scholze.

Daniel Litt: *Integral Aspects of Fundamental Groups*

We study the Galois action on the pro-unipotent fundamental group of a variety, with special attention to how this action interacts with various integral structures. As an application we prove the following theorem: Let X be a normal complex algebraic variety and p a prime. Then there exists $N = N(X, p)$ such that any non-trivial, irreducible representation of $\pi_1(X)$, which arises from geometry, is non-trivial $\mod p^N$.

Fabien Morel: *Etale and motivic variation on Smith's theory on action of finite cyclic groups*

In this talk, after recalling some facts on Smith's theory in classical topology, I will present some analogues in algebraic geometry using étale cohomology first, and I will also "prove" that the multiplicative group $\mathbb{G}_m$ is "cyclic of order 2" and then give analogues in $\mathbb{A}^1$-homotopy theory and motives of Smith's theory concerning actions of $\mathbb{G}_m$.

Baptiste Morin: *On ζ-values of arithmetic schemes*

We give a conjectural description of vanishing orders and special values of zeta functions of arithmetic schemes in terms of Weil-étale cohomology. We also try to explain some connections with Deninger's conjectured cohomology. This is joint work with Matthias Flach.

Paul Arne Østvaer: *Motivic Landweber exact theories and etale cohomology*

We will discuss an etale hyperdescent theorem for Bott inverted motivic Landweber exact theories, e,g., algebraic cobordism and algebraic K-theory. This is achieved by amplifying the effects of the Beilinson-Lichtenbaum conjecture in the motivic slice filtration. Joint work in progress with Elden Elmanto, Marc Levine and Markus Spitzweck.

Tomer Schlank: *Homotopical Obstructions and the unramified Inverse Galois problem*

Given a number field K, the unramified Inverse Galois problem Is concerned with the question which finite groups G can be realized as Galois groups of Galois unramified extensions L/K. The two main ways to attack the problem is by using class field theory (to analyze solvable extensions) and discriminant bounds (to analyze Fields K of small discriminant). The goal of this talk is to show how using homotopical methods one can get results in the non-solvable case with no bound on the discriminant. We will begin by describing a general method to obtain homotopy theoretical obstructions to problems in Galois theory called "Embedding problems". Then we will explain how to employ these obstructions to study the unramified inverse Galois problem. Specifically, using these obstructions on embedding problems with a non-solvable kernel, we'll give an example of an infinite family of groups $\{G_i\}i$ together with an infinite family of quadratic number fields such that for any number field K in this family, the maximal solvable quotient of G_i is realizable as an unramified Galois group over K; but G_i itself is not. This is a joint work with Magnus Carlson.

Johannes Schmidt: *Gabber's geometric presentation lemma over henselian discrete valuation rings*

Gabber's geometric presentation lemma over (infinite) fields was originally used to prove an analogue of Gersten's conjecture for étale cohomology of smooth varieties. A second prominent application was the proof of Morel's connectivity theorem in $\mathbb{A}^1$-homotopy theory over fields. In the talk I will present a version of the presentation lemma over henselian discrete valuation rings with infinite residue fields. Further I will discuss the corresponding two applications over such rings. This is joint work with Florian Strunk.

Florian Strunk: *Algebraic K-theory and descent for blow-ups I*

It is well-known that special cases of descent along blow-ups for algebraic K-theory play an important role for calculating K-groups. For example Thomason proved descent for blow-ups in regularly embedded centers. However in general descent for K-theory of blow-ups does not hold. M. Morrow suggested a potential solution to this problem by considering pro-K-groups of infinitesimal thickenings. We prove such a descent statement in complete generality. As an application, we explain how this implies Weibel's conjecture on the vanishing of negative K-groups. This is joint work with G. Tamme.

Kirsten Wickelgren: *Motivic Euler numbers and an arithmetic count of the lines on a cubic surface*

A celebrated 19th century result of Cayley and Salmon is that a smooth cubic surface over the complex numbers contains exactly 27 lines. Over the real numbers, it is a lovely observation of Finashin–Kharlamov and Okonek–Teleman that while the number of real lines depends on the surface, a certain signed count of lines is always 3. We extend this count to an arbitrary field using an Euler number in $\mathbb{A}^1$-homotopy theory. The resulting count is valued in the Grothendieck–Witt group of non-degenerate symmetric bilinear forms. This is joint work with Jesse Kass.