SOME REMARKS ON THE RESNIKOFF-SALDAÑA CONJECTURE

SOUMYA DAS AND WINFRIED KOHNEN

Abstract. We give some (weak) evidence towards the Resnikoff-Saldaña conjecture on the Fourier coefficients of a degree 2 Siegel cusp form, by proving it for a restricted infinite set of its Fourier coefficients.

1. Introduction

Fourier coefficients of modular forms are a central topic of study in the theory of automorphic forms. The Fourier coefficients $a(f,n)$ of an elliptic cuspidal eigenform f belonging to the space of cusp forms S_k^1 on $SL(2,\mathbf{Z})$ of weight k, if properly normalized, are the eigenvalues of some canonically defined Hecke operators on S_k^1. Deligne's deep theorem (previously the Ramanujan-Petersson conjecture) states that

$$|a(f,n)| \leq \sigma_0(n) n^{(k-1)/2},$$

where $\sigma_0(n)$ denotes the number of divisors of n.

In the case of Siegel cusp forms, which are holomorphic cusp forms on $Sp(n,\mathbf{Z})$, the situation is different. Although a Siegel eigenform F of weight k has a Fourier expansion, the Fourier coefficients are in no straightforward way related to the eigenvalues of the Hecke operators. This is apparent from the fact that even though the Ramanujan-Petersson conjecture for cusp forms on $Sp(2,\mathbf{Z})$ that are orthogonal to the space of the Saito-Kurokawa lifts has been proved by R. Weissauer ([13]), the corresponding conjecture on the Fourier coefficients (known as the Resnikoff-Saldaña (RS) conjecture) is still far from being resolved. Let us denote by S_k^2 the space of Siegel cusp forms of integral weight k for $Sp(2,\mathbf{Z})$. Then the RS conjecture states that for $F \in S_k^2$ with Fourier coefficients $a(F,T)$

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(T being a positive, half-integral 2×2 symmetric matrix) and any $\varepsilon > 0$,

$$a(F,T) \ll_{F,\varepsilon} \det(T)^{k/2-3/4+\varepsilon}. \tag{1.1}$$

Curiously, although some motivation for it was given in [7], the RS conjecture is not known to be true even for a *single* non-zero cusp form F. There are examples however where (1.1) does not hold, namely the Saito-Kurokawa (S-K) lifts. It is believed that apart from these examples, the RS conjecture should be *generically* true. In fact in section 4, we formulate a slightly modified version of the RS conjecture, which we call RS(*), for the S-K lifts.

In this short article, we prove that for any fixed negative fundamental discriminant $-D_0$, the RS conjecture holds for the coefficients $a(F, mT_0)$ for all $m \geq 1$ (the "radial Fourier coefficients" of $F \in S_k^2$) associated to any positive, even-integral 2×2 symmetric matrix T_0 with $\det(T_0) = D_0$ for F that is orthogonal to all S-K lifts. See Theorem 3.1(i) in section 3. Here the proof follows from one of Andrianov's identities (see (3.1)) relating the Fourier coefficients of an eigenform with its eigenvalues together with Weissauer's proof of the Ramanujan-Petersson conjecture [13].

From the same identity we deduce lower bounds for the "radial" Fourier coefficients $a(F, pT_0)$ comparable with the RS conjecture, where p ranges over a set of primes of positive natural density. See Theorem 3.1(ii). For this, we use the recent results on non-vanishing of "fundamental" Fourier coefficients of a cusp form (see [12]) and lower bounds on eigenvalues (see [2]) of degree 2 Siegel cusp forms. It would be interesting to extend this result to arbitrary cusp forms in the orthogonal complement of the space of the Saito-Kurokawa lifts, which we feel may not be straightforward.

In section 4, we deal with the space of S-K lifts. Even though forms in the space spanned by the S-K lifts are known to violate the RS conjecture, they are supposed to satisfy a modified version of it, which we call RS(*) or RS(*) bound, see (4.1). It is then easy to see that RS(*) is a direct consequence of the Ramanujan-Petersson conjecture for certain half-integral weight cusp forms. Finally, in Theorem 4.1, we show that RS(*) is true for the "radial" Fourier coefficients $a(F, mT_0)$ of a S-K lift F with T_0 as above. We also prove lower bounds in the spirit of Theorem 3.1(ii) for the S-K lifts comparable with the RS(*) bound.

We note here, following a suggestion by the referee that our result, namely Theorem 3.1(i), Theorem 4.1(i) are somwwhat reminiscent of how the problem of establishing the equidistribution of Heegner points of discriminant D on $SL_2(\mathbf{Z})\backslash\mathbf{H}$ ($D \to -\infty$) is essentially equivalent to a subconvexity problem when D traverses a sequence of fundamental discriminants (see [4]); but redices to any nontrivial bound for the Hecke eigenvalues of Maass forms on $SL(2,\mathbf{Z})\backslash\mathbf{H}$ when $D = D_0 m^{2n}$ for some fixed fundamental discriminant D_0 and some increasing power m^{2n}.

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2. Brief review of basic facts about Siegel modular forms

For basic facts about Siegel modular forms we refer to [6]. The Siegel modular group of degree 2 is defined as

$$Sp(2,\mathbf{Z}) := \left\{ M \in M_{4\times 4}(\mathbf{Z}) \mid M^t J M = J \right\}, \ J := \begin{pmatrix} 0 & -I_{2\times 2} \\ I_{2\times 2} & 0 \end{pmatrix}.$$

The Siegel upper half space, denoted by $\mathbf{H}_2$ is defined by

$$\mathbf{H}_2 := \{Z \in M_{2\times 2}(\mathbf{C}) \mid Z = Z^t, \ (Z - \overline{Z})/2i > 0\}.$$

It is well known that $Sp(2,\mathbf{R})$ acts on $\mathbf{H}_2$ by

$$Z \mapsto \gamma \cdot Z := (AZ+B)(CZ+D)^{-1}, \quad \gamma = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in Sp(2,\mathbf{R}).$$

The space of Siegel modular forms of weight k and degree 2 consist of holomorphic functions $F\colon \mathbf{H}_2 \to \mathbf{C}$ such that $F(\gamma\cdot Z) = \det(CZ+D)^k F(Z)$ for all $\gamma = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in Sp(2,\mathbf{Z})$. Such a function is called a cusp form if it has a Fourier expansion of the form

$$F(Z) = \sum_{T \in \Lambda_2^+} a(F,T)\, e\left(\mathrm{tr}\ TZ\right),$$

where Λ_2^+ denotes the set of all positive, even-integral 2×2 symmetric matrices, i.e.,

$$\Lambda_2^+ = \{T = T' \in M_{2\times 2}(\mathbf{Z}) \mid T_{ii} \in 2\mathbf{Z},\ T_{ij} \in \mathbf{Z};\ T > 0\}.$$

Further, $e(z) := \exp(\pi i z)$, and tr A denotes the trace of the matrix A.

We denote the space of degree 2 Siegel cusp forms of weight k for the Siegel modular group $Sp(2,\mathbf{Z})$ by S_k^2. For $n \in \mathbf{N}$, one defines the Hecke operator $T(n)$ on S_k^2 by

$$T(n)F = \sum_{\gamma \in Sp(2,\mathbf{Z})\backslash \Delta_{2,n}} F \mid_k \gamma,$$

where $\Delta_{2,n}$ is the set of integral symplectic similitudes of size 4 and scale n:

$$\Delta_{2,n} := \{M \in M_{4\times 4}(\mathbf{Z}) \mid M^t J M = nJ\}.$$

Here we have denoted by $|_k\ \gamma$ the usual action of $Sp(2,\mathbf{R})$ on holomorphic functions $F\colon \mathbf{H}_2 \to \mathbf{C}$:

$$(F \mid_k \gamma)(Z) := (\det \gamma)^{k/2} \det(CZ + D)^{-k} F\left((AZ+B)(CZ+D)^{-1}\right).$$

Let now $F \in S_k^2$ be an eigenfunction with $T(n)F = \lambda_F(n)F$ for all n. Then it is well known that the $\lambda_F(n)$ are real and multiplicative: if $(m,n) = 1$, then $\lambda_F(mn) = \lambda_F(m)\lambda_F(n)$. To this data, one attaches several L-functions, e.g., the standard zeta function and the spinor zeta function $Z_F(s)$.

The function $Z_F(s)$ admits the following Euler product:

$$Z_F(s) = \prod_p Z_{F,p}(s), \quad \text{where } Z_{F,p}(s) = \prod_{1\le i\le 4} (1 - \beta_{i,p} p^{-s})^{-1}. \tag{2.1}$$

Here $\beta_{1,p} := \alpha_{0,p}, \beta_{2,p} := \alpha_{0,p}\alpha_{1,p}, \beta_{3,p} := \alpha_{0,p}\alpha_{2,p}, \beta_{4,p} := \alpha_{0,p}\alpha_{1,p}\alpha_{2,p}$, and the complex numbers $\alpha_{0,p}, \alpha_{1,p}, \alpha_{2,p}$ are the p-Satake parameters of F. We will drop the suffix p when there is no confusion. One has

$$\alpha_0^2 \alpha_1 \alpha_2 = p^{2k-3}, \tag{2.2}$$

and by the Ramanujan–Petersson conjecture (now a theorem due to Weissauer),

$$|\alpha_1| = |\alpha_2| = 1. \tag{2.3}$$

For the well-known analytic properties of $Z_F(s)$ we refer the reader to [1].

The space S_k^2 has a basis consisting of simultaneous eigenfunctions of all the $T(n)$. The space of Saito–Kurokawa lifts, denoted by S_k^*, is the space generated by the lifts of cuspidal Hecke eigenforms of full level and weight $2k-2$, with k even. More precisely, let k be even and $f \in S_{2k-2}$ be a normalized Hecke eigenform. Then there exists (upto scalars) a unique $F_f \in S_k^2$ such that the spinor zeta function $Z_{F_f}(s)$ of F factorizes as (see [5])

$$Z_{F_f}(s) = \zeta(s-k+2)\zeta(s-k+1)L_f(s),$$

where $L_f(s)$ is the Hecke L-function attached to f.

We recall the following formula for the "radial" Fourier coefficients of F_f. Let $a(g,n)$ be the Fourier coefficients of the Shimura-Shintani lift g of weight $k-1/2$ of the eigenform $f \in S_{2k-2}$.

Lemma 2.1 ([3], [5]). *For any $n \geq 1$, one has*

$$a(F_f, nT_0) = a(g, D_0) \prod_{p^\nu || n} \left(\sum_{\mu=0}^{\nu} p^{(\nu-\mu)(k-1)} \left(a(f, p^\mu) - p^{k-2} \left(\frac{-D_0}{p} \right) a(f, p^{\mu-1}) \right) \right).$$

Here $p^\nu || n$ means that p^ν exactly divides n and we set $a(f,n) = 0$ if n is not an integer.

3. Statement of the theorem and proof

Before stating the theorems, we briefly recall how the Saito-Kurokawa (S-K) lifts fail to satisfy the RS conjecture. In fact they do conjecturally satisfy a bound similar to that in the RS conjecture, see section 4.

Suppose that $-D_0$ is a negative fundamental discriminant and $T_0 \in \Lambda_2^+$ has $\det(T_0) = D_0$. It was observed in [9], that infinitely many of the Fourier coefficients $a(F, mT_0)$ $(m \geq 1)$ of a S-K lift F do not satisfy the RS conjecture. This essentially follows from the fact that $Z_F(s)$ has a pole at $s = k$.

The next theorem can be viewed as a converse to the above phenomenon. Let us denote by $\mathscr{D}_{\mathscr{T}}$ the set

$$\mathscr{D}_{\mathscr{T}} := \{(D_0, T_0) \mid -D_0 \text{ fund. disc.}, \ T_0 \in \Lambda_2^+ \text{ such that } \det(T_0) = D_0\}.$$

Theorem 3.1. *Let $F \in S_k^2$ be a cusp form which (in case k is even) that is in the orthogonal complement of S_k^*.*

(i) *Then for all $(D_0, T_0) \in \mathscr{D}_{\mathscr{T}}$, $m \geq 1$ and $\varepsilon > 0$,*

$$a(F, mT_0) \ll_{F, D_0, \varepsilon} m^{k-3/2+\varepsilon}.$$

(ii) *Let F be an eigenform. There exist $(D_0, T_0) \in \mathscr{D}_{\mathscr{T}}$ such that for a set of primes p with positive lower natural density,*

$$a(F, pT_0) \gg_{F,D_0} p^{k-3/2}.$$

Here, for a subset A of the set of all primes $\mathcal{P}$, lower natural density of A, denoted as $\underline{\delta}_{\mathrm{Nat}}(A)$, is defined by:

$$\underline{\delta}_{\mathrm{Nat}}(A) := \liminf_{X \mapsto \infty} \frac{\#\{p \leq X \mid p \in A\}}{\#\{p \leq X\}}.$$

Proof. Clearly it suffices to prove (i) for eigenforms. We start with a fundamental identity of Andrianov relating the Fourier coefficients of a degree 2 cusp form with its eigenvalues (see [1, Thm. 2.4.1])

$$L_{D_0}(s-k+2, \chi) \sum_{i=1}^{h} \chi(T_i) \sum_{m=1}^{\infty} \frac{a(F, mT_i)}{m^s} = \psi_F(s, \chi) Z_F(s). \tag{3.1}$$

Here $L_{D_0}(s, \chi)$ is the L-function of the ideal class group I of $K := \mathbf{Q}(\sqrt{-D_0})$ corresponding to the character χ of I, and $h := h(-D_0)$ is the class number (the cardinality of I) of K. Further, $\{T_1, \ldots, T_h\}$ are $SL(2, \mathbf{Z})$-inequivalent matrices of determinant D_0 and $\psi_F(s, \chi) := \psi_F^{D_0}(s, \chi)$ is a certain finite Dirichlet series, which in the case that $-D_0$ is a fundamental discriminant is just the constant function given by:

$$\psi_F(s, \chi) := \sum_i \chi(T_i) a(F, T_i). \tag{3.2}$$

Inverting the character sum in (3.1), we get

$$\sum_{m=1}^{\infty} \frac{a(F, mT_j)}{m^s} = \frac{1}{h} \left(\sum_{\chi} \frac{\overline{\chi}(T_j)\ \psi_F(s, \chi)}{L_{D_0}(s-k+2, \chi)} \right) Z_F(s). \tag{3.3}$$

Let us denote the Dirichlet coefficients of the Dirichlet series in braces in the above equation by a_m and that of $Z_F(s)$ by b_m. Then a calculation shows that

$$a_m = \sum_i a(F, T_i) \sum_{\chi} \overline{\chi}(T_j) \chi(T_i) \mu(m) m^{k-2} \sum_{\mathfrak{a}\colon N(\mathfrak{a})=m} \chi(\mathfrak{a}), \tag{3.4}$$

where $N(\mathfrak{a})$ denotes the norm of the ideal $\mathfrak{a}$ and $\mu(\cdot)$ is the Möbius function.

Let $\mathscr{N}(m)$ denote the number of integral ideals of K with norm m. Then it is well known that

$$\mathscr{N}(m) = \sum_{d|m} \chi_{-D_0}(d),$$

where $\chi_{-D_0} = \chi_{-D_0,K}$ is the Dirichlet character associated to K (in fact the formula follows from the relation $\zeta_K(s) = \zeta_{\mathbf{Q}}(s)L(s, \chi_{-D_0})$).

Thus from (3.4), using the Hecke bound for the coefficients $a(F, T_i)$, we get the following estimate for a_m:

$$a_m \ll_F h^2 m^{k-2} D_0^{k/2} \mathscr{N}(m) \ll_{F,D_0,\varepsilon} h^2 m^{k-2+\varepsilon}. \tag{3.5}$$

Moreover, estimates for the quantities b_m are known from the Ramanujan-Petersson conjecture for F, proved by R. Weissauer ([13]), see (2.1).

Since the local Euler factor $Z_{F,p}(s)$ is of degree 4, it follows from (2.2) and (2.3) (by comparing with $\zeta_{\mathbf{Q}}^4(s)$) that

$$b_m \leq d_4(m) m^{k-3/2} := \#\{(a,b,c,d) \colon abcd = m\} \cdot m^{k-3/2} \ll_\varepsilon m^{k-3/2+\varepsilon}. \tag{3.6}$$

Thus from (3.4) and (3.6), we get

$$\begin{aligned} a(F, mT_j) = \frac{1}{h} \sum_{d|m} a_d b_{m/d} &\ll_{F,D_0,\varepsilon} h m^{k-3/2+\varepsilon} \sum_{d|m} d^{-1/2} \\ &\ll_{F,D_0,\varepsilon} m^{k/2-3/4+\varepsilon}. \end{aligned} \tag{3.7}$$

This proves part (i) of the theorem. For part (ii), we again look at (3.7). We get for a prime p,

$$a(F, pT_j) = \frac{1}{h}(a_1 b_p + a_p b_1) = \frac{1}{h}(\lambda_F(p) a(F, T_j) + a_p)). \tag{3.8}$$

We now appeal to two results. First, to a result of A. Saha [12], which states that there exist a square–free negative fundamental discriminant $-D_0$ (odd and square–free) such that $a(F, T_j) \neq 0$ for some j. We fix such a D_0 and j. Secondly, recall from [2] that on a set $\mathcal{S}_c$ of positive density, one has (for any $0 < c < 1/4$)

$$\lambda_F(p) > c \cdot p^{k-3/2}.$$

Taking into account the estimate of a_p from (3.5) and the above remarks, we easily get (ii) for large primes $p \in \mathcal{S}_c$ from (3.8). This completes the proof of Theorem 3.1(ii). □

Remark 3.2. In [8], estimates of eigenvalues of cuspidal eigenforms were obtained from those of the Fourier coefficients by considering in detail the action of the Hecke operators $T(p)$ on the Fourier expansion of a cusp form. In this paper we do the opposite.

4. The case of Saito-Kurokawa lifts

We now state RS(*), a modified version of RS conjecture, which the elements of S_k^* are expected to satisfy. Let $G \in S_k^*$, $T \in \Lambda_2^+$ and $\varepsilon > 0$. Then

$$a(G,T) \ll_{F,\varepsilon} \det(T)^{k/2-1/2+\varepsilon}. \tag{4.1}$$

If h is the eigenform of weight $k - \frac{1}{2}$ in the Kohnen's + space corresponding to G, then it is known that (see [3] for example):

$$a(G,T) = \sum_{d|c_T} d^{k-1} a(h, |\det(T)|/d^2), \tag{4.2}$$

where c_T is the content of T, i.e., the greatest common divisor of the entries of T.

If we assume the Ramanujan-Petersson conjecture for h (see [10]), which states that for $n \geq 1$,

$$a(h,n) \ll_{h,\varepsilon} n^{k/2-3/4+\varepsilon}; \tag{4.3}$$

plugging (4.3) into (4.2) we can get (4.1):

$$\begin{aligned} a(G,T) &\ll \sum_{d|c_T} d^{k-1} \left(\frac{\det(T)}{d^2}\right)^{k/2-3/4+\varepsilon} \\ &\ll \det(T)^{k/2-3/4+\varepsilon} \sum_{d|c_T} d^{\frac{1}{2}-\varepsilon} \\ &\ll \det(T)^{k/2-3/4+\varepsilon} c_T^{\frac{1}{2}+\varepsilon} \ll \det(T)^{k/2-1/2+\varepsilon}. \end{aligned}$$

We now proceed to prove an analogous version of Theorem 3.1 in the case of S-K lifts; towards the (RS*) in mind.

Theorem 4.1. *Let G in S_k^* be arbitrary, and k be even.*

(i) *Then for all* $(D_0, T_0) \in \mathscr{D}_{\mathscr{T}}$, $m \geq 1$ *and* $\varepsilon > 0$,

$$a(G, mT_0) \ll_{F,D_0,\varepsilon} m^{k-1+\varepsilon}.$$

(ii) *There exist* $(D_0, T_0) \in \mathscr{D}_{\mathscr{T}}$ *such that all primes* p,

$$a(F, pT_0) \gg_{F,D_0} p^{k-1}.$$

Proof. Clearly it suffices to prove (i) for eigenforms. Let us recall the formula for the 'radial' Fourier coefficients $a(G, nT)$ of G from Lemma 2.1. Applying Deligne's bound

$$a(f, n) \ll_\varepsilon n^{k-3/2+\varepsilon} \quad (\varepsilon > 0)$$

on the Fourier coefficients of the eigenform $f \in S_{2k-2}$ corresponding to G we get:

$$\begin{aligned} a(G, nT) &\ll_{D_0,\varepsilon} \prod_{p^\nu \| n} \left(\sum_{\mu=0}^{\nu} p^{(\nu-\mu)(k-1)} \left(p^{(k-\frac{3}{2})\mu+\varepsilon} + p^{k-2+(k-\frac{3}{2})(\mu-1)+\varepsilon} \right) \right) \\ &\ll_{D_0,\varepsilon} \prod_{p^\nu \| n} p^{\nu(k-1)+\varepsilon} \left(\sum_{\mu=0}^{\nu} p^{-\frac{\mu}{2}} \right) \\ &\ll_{D_0,\varepsilon} n^{k-1+\varepsilon}. \end{aligned}$$

This proves (i).

For (ii), let $G \in S_k^*$ be arbitrary, and write for complex numbers u_i

$$G = \sum_{i=1}^{\mu} u_i G_i,$$

where $\mu = \dim S_{2k-2}$ and the eigenforms G_i are lifts of the corresponding normalized eigenforms $f_i \in S_{2k-2}$. Further, let g_i be a Shimura-Shintani lift of f_i.

According to the Lemma 2.1 we have,

(4.4)

$$a(G, pT_0) = \sum_{i=1}^{\mu} \left(u_i a(g_i, D_0) p^{k-1} + u_i a(g_i, D_0) a(f_i, p) + u_i a(g_i, D_0) \left(\frac{-D_0}{p} \right) p^{k-2} \right).$$

According to [12], for the (non-zero) half-integral weight cusp form g in the Kohnen's + space defined by

$$g = \sum_{i=1}^{\mu} u_i g_i,$$

there exist at least one odd, square–free D_0 such that, $a(g, D_0) \neq 0$.

With this choice of D_0, we have from (4.4) (using Deligne's bound for the integral weight forms and Hecke's bound in the half-integral weight case) that

$$\begin{aligned}|a(G,pT_0)| &\geq |a(g,D_0)|p^{k-1} - \sum_{i=1}^{\mu}\big(|u_i a(g_i,D_0)a(f_i,p)| + |u_i|p^{k-2}\big)\\ &\geq |a(g,D_0)|p^{k-1} - (2p^{(k-1)/2}D_0^{k/2-3/4} + p^{k-2})(\sum_{i=1}^{\mu}|u_i|)\\ &\gg_{G,D_0} p^{k-1}.\end{aligned}$$

□

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Department of Mathematics, Indian Institute of Science, Bangalore – 560012, India.

E-mail address: somu@math.iisc.ernet.in

Mathematisches Institut, Ruprecht-Karls-Universitat Heidelberg, D – 69120 Heidelberg, Germany.

E-mail address: winfried@mathi.uni-heidelberg.de