BERNSTEIN CENTER OF SUPERCUSPIDAL BLOCKS

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Abstract. Let $\mathbf{G}$ be a tamely ramified connected reductive group defined over a non-archimedean local field k. We show that the Bernstein center of a tame supercuspidal block of $\mathbf{G}(k)$ is isomorphic to the Bernstein center of a depth zero supercuspidal block of $\mathbf{G}^0(k)$ for some twisted Levi subgroup of $\mathbf{G}^0$ of $\mathbf{G}$.

1. Introduction

Let $\mathbf{G}$ be a connected reductive group defined over a non archimedean local field k. Assume that $\mathbf{G}$ splits over a tamely ramified extension k^t of k. We will denote the group of k-rational points of $\mathbf{G}$ by G and likewise for other algebraic groups. In [8], Jiu-Kang Yu gives a very general construction of a class of supercuspidal representations of G which he calls *tame*. A tame supercuspidal representation $\pi = \pi_\Sigma$ of G is constructed out of a depth zero supercuspidal representation π_0 of G^0 and some additional data, where $\mathbf{G}^0$ is a *twisted* Levi subgroup of $\mathbf{G}$. By twisted, we mean that $\mathbf{G}^0 \otimes k^t$ is a Levi factor of a parabolic subgroup of $\mathbf{G} \otimes k^t$. The additional data, together with $\mathbf{G}^0$ and π_0 is what we are denoting by Σ in the notation π_Σ. In [4], Kim showed that under certain hypothesis, which are met for instance when the residue characteristic is large, these tame supercuspidals exhaust all the supercuspidals of G.

The depth zero supercuspidal π_0 of G^0 is compactly induced from (K^0, ϱ_0) where K^0 is a compact mod center open subgroup of G^0 and ϱ_0 is a representation of K^0. The constructed representation π_Σ is compactly induced from (K, ϱ), where K is a compact mod center open subgroup of G containing K^0 and ϱ is a representation of K. The representation ϱ is of the form $\varrho_0 \otimes \kappa$, where ϱ_0 is seen as a representation of K by extending from K^0 "trivially" (see [8, Sec. 4]) and κ is a representation of K constructed out of the part of Σ which is independent of ϱ_0.

Let $\mathfrak{Z}^\pi$ (resp. $\mathfrak{Z}_0^{\pi_0}$) denote the *Bernstein center* of the *Bernstein block* (see Section 4 for these terms) of G (resp. G^0) containing π (resp. π_0). Under certain hypothesis $C(\overrightarrow{\mathbf{G}})$ [3, Page 47], we show that:

Theorem. $\mathfrak{Z}^\pi \cong \mathfrak{Z}_0^{\pi_0}$. *Thus, the Bernstein center of a tame supercuspidal block of G is isomorphic to the Bernstein center of a depth zero supercuspidal block of a twisted Levi subgroup of G.*

Let $\mathcal{H}(G, \varrho)$ (resp. $\mathcal{H}(G^0, \varrho_0)$) denote the Hecke algebra of the type constructed out of (K, ϱ) (resp. (K^0, ϱ_0)) (see [1, Sec. 5.4]). As a consequence of the above theorem, we obtain

$$Z(\mathcal{H}(G, \varrho)) \cong Z(\mathcal{H}(G^0, \varrho_0)).$$

Under certain conditions on π ([1, Sec. 5.5]) which are satisfied quite often, for instance whenever π is generic, the Hecke algebra $\mathcal{H}(G, \varrho)$ is commutative. Assuming these conditions, in Corollary 11 we prove a Conjecture of Yu [8, Conj. 0.2], which is a special case of his more general conjecture [8, Conj. 17.7].

2. Notations

Throughout this article, k denotes a non-archimedean local field. For an algebraic group $\mathbf{G}$ defined over k, we will denote its k-rational points by G. We will follow standard abuses of notation and terminology and refer, for example, to parabolic subgroups of G in place of k-points of k-parabolic subgroups of $\mathbf{G}$. Center of $\mathbf{G}$ will be denoted by $\mathbf{Z_G}$. The category of smooth representations of G will be denoted by $\mathfrak{R}(G)$.

3. Yu's construction [8]

Let $\mathbf{G}$ be a connected reductive group defined over a non-archimedean local field k. A twisted k-Levi subgroup $\mathbf{G}'$ of $\mathbf{G}$ is a reductive k-subgroup such that $\mathbf{G}' \otimes_k \bar{k}$ is a Levi subgroup of $\mathbf{G} \otimes_k \bar{k}$. Yu's construction involves the notion of a generic $\mathbf{G}$-datum. It is a quintuple $\Sigma = (\overrightarrow{\mathbf{G}}, y, \overrightarrow{r}, \overrightarrow{\phi}, \rho)$ satisfying the following:

(1) $\overrightarrow{\mathbf{G}} = (\mathbf{G}^0 \subsetneq \mathbf{G}^1 \subsetneq \ldots \subsetneq \mathbf{G}^d = \mathbf{G})$ is a tamely ramified twisted Levi sequence such that $\mathbf{Z}_{\mathbf{G}^0}/\mathbf{Z_G}$ is anisotropic.

(2) y is a point in the extended Bruhat-Tits building of $\mathbf{G}^0$ over k.

(3) $\overrightarrow{r} = (r_0, r_1, \cdots, r_{d-1}, r_d)$ is a sequence of positive real numbers with $0 < r_0 < \cdots < r_{d-2} < r_{d-1} \leq r_d$ if $d > 0$, $0 \leq r_0$ if $d = 0$.

(4) $\overrightarrow{\phi} = (\phi_0, \cdots, \phi_d)$ is a sequence of quasi-characters, where ϕ_i is a G^{i+1}-*generic quasi-character* [8, Sec. 9] of G^i; ϕ_i is trivial on G^i_{y,r_i+}, but nontrivial on G^i_{y,r_i} for $0 \leq i \leq d-1$. If $r_{d-1} < r_d$, ϕ_d is nontrivial on G^i_{y,r_d} and trivial on G^d_{y,r_d+}. Otherwise, $\phi_d = 1$. Here $G^i_{y,r}$ denote the filtration subgroups of the parahoric at y defined by Moy-Prasad (see [6, Sec. 2.6]).

(5) ρ is an irreducible representation of $G^0_{[y]}$, the stabilizer in G^0 of the image $[y]$ of y in the reduced building of $\mathbf{G}^0$, such that $\rho|G^0_{y,0+}$ is isotrivial and $c\text{-Ind}_{G^0_{[y]}}^{G^0} \rho$ is irreducible and supercuspidal.

Let $K^0 = G^0_{[y]}$ and $K^i = G^0_{[y]} G^1_{y,s_0} \cdots G^i_{y,s_{i-1}}$ where $s_j = r_j/2$ for $i = 1, \ldots, d$. In [8, Sec. 11], Yu constructs certain representation κ of $K^d = K^d(\Sigma)$ which is independent of ρ and constructed only out of $(\vec{\mathbf{G}}, y, \vec{r}, \vec{\phi})$. Extend ρ "trivially" to a representation of K^d (see [8, Sec. 4]) and write $\rho_\Sigma := \rho \otimes \kappa$.

Theorem 1 (Yu)**.** *$\pi_\Sigma := c\text{-Ind}_{K^d}^G \rho_\Sigma$ is irreducible and thus supercuspidal.*

The following theorem of Kim [4] says that under certain hypothesis (which are met for instance when the residue characteristic is sufficiently large), the representations π_Σ for various generic $\mathbf{G}$-datum Σ exhaust all the supercuspidal representations of G.

Theorem 2 (Ju-Lee Kim)**.** *Suppose the hypothesis* (H k)*,* (HB)*,* (HGT) *and* (H $\mathcal{N}$) *in* [4] *are valid. Then all the supercuspidal representations of G arise through Yu's construction.*

In [3, Theorem 6.6, 6.7] under certain hypothesis denoted by $C(\vec{\mathbf{G}})$ [3, Page 47], Hakim and Murnaghan determine when two supercuspidal representations are equivalent:

Theorem 3 (Hakim-Murnaghan)**.** *Let $\Sigma = (\vec{\mathbf{G}}, y, \vec{r}, \vec{\phi}, \rho)$ and $\Sigma' = (\vec{\mathbf{G}}', y', \vec{r}', \vec{\phi}', \rho')$ be two generic G-data. Set $\phi = \Pi_{i=1}^d \phi_i | G^0$, $\phi' = \Pi_{i=1}^{d'} \phi_i' | G^{0\prime}$, $\pi_0 = c\text{-Ind}_{G^0_{[y]}}^{G^0} \rho$ and $\pi_0' = c\text{-Ind}_{G^{0\prime}_{[y']}}^{G^{0\prime}} \rho'$. Then $\pi_\Sigma \cong \pi_{\Sigma'}$ if and only if there exists $g \in G$ such that $K^d(\Sigma) = {}^g K^{d'}(\Sigma')$ and $\rho_\Sigma = {}^g \rho_{\Sigma'}$ if and only if $G^0 = {}^g G^{0\prime}$ and $\pi_0 \otimes \phi \cong^g (\pi_0' \otimes \phi')$.*

4. Bernstein decomposition

Let $X_k(\mathbf{G}) = \mathrm{Hom}(\mathbf{G}, \mathbb{G}_m)$, the lattice of k-rational characters of $\mathbf{G}$. Let

$$^\circ G := \{g \in G : \mathrm{val}_k(\chi(g)) = 0, \forall \chi \in X_k(\mathbf{G})\}.$$

In [5, Section 7], Kottwitz defined a functorial homomorphism $\kappa_G' : G \twoheadrightarrow X_*(\mathbf{Z_G})_{I_k}^{\mathrm{Fr}}$. Here $X_*(\mathbf{Z_G})$ denotes the cocharacter lattice of $\mathbf{Z_G}$, $(\cdot)^{\mathrm{Fr}}$ (resp. $(\cdot)_{I_k}$) denotes taking invariant (resp. coinvariant) with respect to Frobenius Fr (resp. inertia subgroup I_k). The map κ_G' induces a functorial surjective map:

$$\kappa_G : G \twoheadrightarrow X_*(\mathbf{Z_G})_{I_k}^{\mathrm{Fr}} / \text{torsion} \tag{4.1}$$

and $\ker(\kappa_G)$ is precisely $^\circ G$ (see [2, Sec. 3.3.1]).

Let $X_{\mathrm{nr}}(G) := \mathrm{Hom}(G/{}^\circ G, \mathbb{C}^\times)$ denote the group of *unramified characters* of G. For a smooth representation π of G, the representations $\pi \otimes \chi$, $\chi \in X_{\mathrm{nr}}(G(k))$ are called the *unramified twists* of π.

Consider the collection of all cuspidal pairs (L, σ) consisting of a Levi subgroup L of G and an irreducible cuspidal representation σ of L. Define an equivalence relation $\sim$ on the class of all cuspidal pairs by

$$(L, \sigma) \sim (M, \tau) \text{ if } {}^g L = M \text{ and } {}^g \sigma \cong \tau\nu,$$

for some $g \in G$ and some $\nu \in X_{\rm nr}(M)$. Write $[L,\sigma]$ for the equivalence class of (L,σ) and $\mathfrak{B}(G)$ for the set of all equivalence classes. The set $\mathfrak{B}(G)$ is called the *Bernstein spectrum* of G. We say that a smooth irreducible representation π has *inertial support* $s := [L,\sigma]$ if π appears as a subquotient of a representation parabolically induced from some element of $\mathfrak{s}$. Define a full subcategory $\mathfrak{R}^{\mathfrak{s}}(G)$ of $\mathfrak{R}(G)$ as follows: a smooth representation π belongs to $\mathfrak{R}^{\mathfrak{s}}(G)$ iff each irreducible subquotient of π has inertial support $\mathfrak{s}$. The categories $\mathfrak{R}^{\mathfrak{s}}(G), \mathfrak{s} \in \mathfrak{B}(G)$, are called the *Bernstein Blocks* of G.

Theorem 4 (Bernstein)**.** *We have*

$$\mathfrak{R}(G) = \prod_{\mathfrak{s}\in\mathfrak{B}(G)} \mathfrak{R}^{\mathfrak{s}}(G).$$

Definition 5. The endomorphism ring of the identity functor of $\mathfrak{R}(G)$ (resp. $\mathfrak{R}(G)^{\mathfrak{s}}$) is called the *Bernstein center* of $\mathfrak{R}(G)$ (resp. $\mathfrak{R}(G)^{\mathfrak{s}}$).

The following result of Roche [7, Theorem 1.10.3.1] relates the Bernstein center of a block with the center of the corresponding Hecke algebra.

Theorem 6 (Roche)**.** *Let e be an idempotent in the Hecke algebra $\mathcal{H} = \mathcal{H}(G)$. View $\mathcal{H}$ as a smooth G-module via the left regular representation, and write $e = \sum_{\mathfrak{s}\in\mathfrak{S}} e_{\mathfrak{s}}$ according to the Bernstein decomposition $\mathcal{H} = \bigoplus_{\mathfrak{s}\in\mathfrak{S}_e} \mathcal{H}_{\mathfrak{s}}$. Let $\mathfrak{S}_e = \{\mathfrak{s} \in \mathfrak{B}(G) : e_{\mathfrak{s}} \neq 0\}$ and $\mathfrak{Z}(G)^{\mathfrak{S}_e} = \prod_{\mathfrak{s}\in\mathfrak{S}_e} \mathfrak{Z}(G)^{\mathfrak{s}}$, where $\mathfrak{Z}(G)^{\mathfrak{s}}$ is the Bernstein center of the block $\mathfrak{R}(G)^{\mathfrak{s}}$. Let $\mathcal{Z}(e\mathcal{H}e)$ denote the center of the algebra $e\mathcal{H}e$.*

Then the map $z \mapsto z(e)$ defines an algebra isomorphism $\mathfrak{Z}(G)^{\mathfrak{S}_e} \tilde{\to} \mathcal{Z}(e\mathcal{H}e)$.

5. Main result

We use the notations of Section 3. So $\mathbf{G}$ is a connected reductive group over k, $\Sigma = (\overrightarrow{\mathbf{G}}, y, \overrightarrow{r}, \overrightarrow{\phi}, \rho)$ is a generic $\mathbf{G}$-datum, $K^0 = G^0_{[y]}$ and $K^i = G^0_{[y]} G^1_{y,s_0} \cdots G^i_{y,s_{i-1}}$ where $s_j = r_j/2$ for $i = 1, \ldots, d$. Then in [8], Yu constructs a representation ρ_Σ of K^d such that $\pi_\Sigma := c\text{-Ind}_{K^d}^G \rho_\Sigma$ is irreducible and thus supercuspidal. The representation $\pi_0 = c\text{-Ind}_{K^0}^{G^0} \rho$ is depth zero supercuspidal. Write ${}^\circ K^d := K^d \cap {}^\circ G$ (resp. ${}^\circ K^0 := K^0 \cap {}^\circ G^0$) and ${}^\circ\rho_\Sigma := \rho_\Sigma | {}^\circ K^d$ (resp. ${}^\circ\rho = \rho | {}^\circ K^0$). Here ${}^\circ G$ is as defined in Section 4. Then $({}^\circ K^d, {}^\circ\rho_\Sigma)$ (resp. $({}^\circ K, {}^\circ\rho)$) is an $\mathfrak{s} := [G, \pi_\Sigma]_G$ (resp. $\mathfrak{s}_0 := [G^0, \pi_0]_{G^0}$) type [8, Corr. 15.3].

Let $\mathfrak{Z}(G)$ (resp. $\mathfrak{Z}(G)^{\mathfrak{s}}$, resp. $\mathfrak{Z}(G^0)^{\mathfrak{s}_0}$) be the Bernstein center of the category $\mathfrak{R}(G)$ (resp. $\mathfrak{R}(G)^{\mathfrak{s}}$, resp. $\mathfrak{R}(G^0)^{\mathfrak{s}_0}$). Assume the hypothesis $C(\overrightarrow{\mathbf{G}})$ in [3, Page 47].

Theorem 7. $\mathfrak{Z}(G)^{\mathfrak{s}} \cong \mathfrak{Z}(G^0)^{\mathfrak{s}_0}$.

Proof. By functoriality of the map (4.1), the inclusion $\mathbf{G}^0 \hookrightarrow \mathbf{G}$ induces a map $\chi \in X_{\rm nr}(G) \mapsto \chi | G^0 \in X_{\rm nr}(G^0)$. For an irreducible representation τ of G, define

$$\mathfrak{S}^G(\tau) = \{\nu \in X_{\rm nr}(G) : \tau\nu \cong \tau\}.$$

Similarly define $\mathfrak{S}^{G^0}(\mu)$ for an irrep μ of G^0. Given $\chi \in X_{\rm nr}(G)$, define a new quintuple $\Sigma_\chi = (\vec{\mathbf{G}}, y, \vec{r}, \vec{\phi}, \rho \otimes (\chi|K^0))$. We have $\pi_\Sigma \otimes \chi \cong c\text{-Ind}_{K^d}^G(\rho \otimes \kappa \otimes (\chi|K^d))$. Since χ is unramified, it follows that $\pi_\Sigma \otimes \chi \cong \pi_{\Sigma_\chi}$. Now if $\chi \in X_{\rm nr}(G)$ is such that $\pi_0 \otimes \chi|G^0 \cong \pi_0$, then it follows from Theorem 3 or directly that $\pi_{\Sigma_\chi} \cong \pi_\Sigma$, i.e., $\pi_\Sigma \otimes \chi \cong \pi_\Sigma$. Conversely suppose $\pi_\Sigma \otimes \chi \cong \pi$. By Theorem 3 ([3, Theorem 6.6 and 6.7]), this is equivalent to (K^d, ρ_Σ) being G-conjugate to $(K^d, \rho_{\Sigma_\chi})$. Let $\rho' = (\rho \otimes \chi|K^0)$. Since $\rho|G^0_{y,0+} = \rho'|G^0_{y,0+}$ is isotrivial, it follows from [8, Prop. 4.4 and 4.1] that we can assume the conjugating element g to be in G^0. Then by Theorem 3, we get $\pi_0 \otimes \phi \cong (\pi_0' \otimes \phi)$ as G^0-representations, where ϕ is as in Theorem 3 and $\pi_0' = c\text{-Ind}_{G^0_{[y]}}^{G^0} \rho'$. It follows that $\pi_0 \otimes \chi|G^0 \cong \pi_0$. Thus we get an injective map $X_{\rm nr}(G)/\mathfrak{S}^G(\pi_\Sigma) \to X_{\rm nr}(G^0)/\mathfrak{S}^{G_0}(\pi_0)$.

Now given $\nu \in X_{\rm nr}(G^0)$, using notation similar to before, write $\Sigma_\nu = (\vec{\mathbf{G}}, y, \vec{r}, \vec{\phi}, \rho \otimes (\nu|K^0))$. Since $({}^\circ K^d, {}^\circ\rho_\Sigma) = ({}^\circ K^d, {}^\circ\rho_{\Sigma_\nu})$ and $({}^\circ K^d, {}^\circ\rho_\Sigma)$ is an $\mathfrak{s}$-type, it follows that $\pi_{\Sigma_\nu} \cong \pi_\Sigma \otimes \chi$ for some $\chi \in X_{\rm nr}(G)$. Then again we have, $\pi_0 \otimes \nu \cong \pi_0 \otimes (\chi|G^0)$, i.e., $(\chi|G^0) - \nu \in \mathfrak{S}^{G^0}(\pi_0)$. This shows that the map $X_{\rm nr}(G)/\mathfrak{S}^G(\pi_\Sigma) \to X_{\rm nr}(G^0)/\mathfrak{S}^{G_0}(\pi_0)$ is also surjective and therefore an isomorphism.

Let $\mathrm{Irr}^{\mathfrak{s}}(G)$ (resp. $\mathrm{Irr}^{\mathfrak{s}_\circ}(G^0)$) denote the isomorphism classes of irreducible elements in $\mathfrak{R}(G)^{\mathfrak{s}}$ (resp. $\mathfrak{R}(G^0)^{\mathfrak{s}_\circ}$). The isomorphism $X_{\rm nr}(G)/\mathfrak{S}^G(\pi_\Sigma) \to X_{\rm nr}(G^0)/\mathfrak{S}^{G_0}(\pi_0)$ induces an isomorphism $\mathrm{Irr}^{\mathfrak{s}}(G) \to \mathrm{Irr}^{\mathfrak{s}_\circ}(G^0)$. It is clear that the later is independent of the choice of π_Σ in $\mathrm{Irr}^{\mathfrak{s}}(G)$. Since $\mathfrak{Z}(G)^{\mathfrak{s}}$ (resp. $\mathfrak{Z}(G^0)^{\mathfrak{s}_0}$) is the ring of regular functions on $\mathrm{Irr}^{\mathfrak{s}}(G)$ (resp. $\mathrm{Irr}^{\mathfrak{s}_\circ}(G^0)$), the Theorem follows. □

Let $\Sigma' = (\vec{\mathbf{G}}', y', \vec{r}', \vec{\phi}', \rho')$ be another generic G-data such that $\pi_\Sigma \cong \pi_{\Sigma'}$. Then arguing as in Theorem 7, we obtain that there is a canonical isomorphism $\mathfrak{Z}(G^0)^{\mathfrak{s}_0} \cong \mathfrak{Z}(G^{0\prime})^{\mathfrak{s}_0'}$. Write $\mathfrak{Z}(G)^{\mathfrak{s}}_0 = \varprojlim \mathfrak{Z}(G^0)^{\mathfrak{s}_0}$ to get a ring independent of the choice of Σ. Then Theorem 7 can be written in a canonical way as:

Theorem 8. *There is a canonical isomorphism* $\mathfrak{Z}(G)^{\mathfrak{s}} \cong \mathfrak{Z}(G)^{\mathfrak{s}}_0$.

For each irreducible object $\tau \in \mathfrak{R}(G)$ and $z \in \mathfrak{Z}(G)$, denote by $\chi_z(\tau)$, the scalar by which z acts on τ. Let $z \in \mathfrak{Z}(G)^{\mathfrak{s}} \mapsto z_0 \in \mathfrak{Z}(G^0)^{\mathfrak{s}_\circ}$ and $\pi \in \mathrm{Irr}^{\mathfrak{s}}(G) \mapsto \pi_0 \in \mathrm{Irr}^{\mathfrak{s}_0}(G^0)$ under the isomorphisms described in Theorem 7.

Corollary 9. $\chi_z(\pi) = \chi_{z_0}(\pi_0)$.

Proof. This follows from [7, Prop. 1.6.4.1] and Theorem 7. □

For an algebra $\mathcal{A}$, denote by $Z(\mathcal{A})$ the center of $\mathcal{A}$. Let $\mathcal{H}(G, \rho_\Sigma)$ (resp. $\mathcal{H}(G^0, \rho)$) denote the Hecke algebra of the type $({}^\circ K^d, {}^\circ\rho_\Sigma)$ (resp. $({}^\circ K^0, {}^\circ\rho)$).

Corollary 10. $Z(\mathcal{H}(G, \rho_\Sigma)) \cong Z(\mathcal{H}(G^0, \rho))$.

Proof. This follows from [7, Theorem 1.10.3.1]. □

Now suppose that π_Σ satisfied the conditions (5.5) of [1]. These conditions are quite frequently satisfied, for instance whenever π_Σ is generic and therefore in particular for $\mathbf{G} = \mathrm{GL}(n, k)$. In that case, $\mathcal{H}(G, \rho_\Sigma)$ is commutative [1, Sec. 5.6]. With this assumption, we get the following corollary which is a special case of Yu's conjecture [8, Conjecture 17.7]:

Corollary 11. $\mathcal{H}(G, \rho_\Sigma) \cong \mathcal{H}(G^0, \rho)$.

Proof. Since $\mathcal{H}(G, \rho_\Sigma)$ is commutative, $Z(\mathcal{H}(G, \rho_\Sigma)) = \mathcal{H}(G, \rho_\Sigma)$. By Corollary 10, we therefore have $\mathcal{H}(G, \rho_\Sigma) \cong Z(\mathcal{H}(G^0, \rho))$. But $\mathcal{H}(G^0, \rho)$ is naturally a subspace of $\mathcal{H}(G, \rho_\Sigma)$ ([8, Theorem 17.9]). It follows that $\mathcal{H}(G, \rho_\Sigma) \cong \mathcal{H}(G^0, \rho)$. □

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