## Errata for:

## Cohomology of Number Fields. Online Edition 2.3, May 2020

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This file lists known mistakes. If you find a mistake not listed below or have a comment, please send me an e-mail: schmidt@mathi.uni-heidelberg.de.

-p. 18, l. 5 The section s must be chosen such that s(1) = 1. This is possible after a translation.

- -p. 18, l. 13 remove 'and A is closed in  $\hat{G}$ '.
- -p. 21, l. -6 (noticed by D. Vogel) replace '§8' by '§9'.
- -p. 23, l. -1 (noticed by D. Vogel) replace '§8' by '§9'.
- -p. 59, Ex. 8 (noticed by C. Lane) The formulation of the exercise is messed up.
- -p. 144, l. 14 replace  ${}_{\ell^m}H^n_{cts}(G,T)$  by  ${}_{\ell^m}H^{n+1}_{cts}(G,T)$ .
- -p. 209 , Ex. 2 (noticed by C. Lane) Add the assumption that the curve in question has genus  $\geq 1$ .
- -p. 219, l. -4 (noticed by K. Koziol) replace 'a Poincaré group' by 'a pro-p Poincaré group'.
- -p. 326, l. -12 (noticed by C. Lane) replace  ${}^{*}N^{ab}_{\mathscr{H}}/G$  by  ${}^{*}N^{ab}_{\mathscr{H}}/\operatorname{Rad}_{G}$ .
- -p. 408, l. 3 (noticed by C. Lane) replace  $T_k$  by  $G_k$ .

-p. 466, Ex. 1 (noticed by C. Lane) The statement of the exercise is false. It becomes correct after replacing  $\mathcal{O}_S^{\times}$  by  $k_S^{\times}$ . (However, then it is a special case of (6.3.4).)

**-p. 484, bottom** (noticed by B. Selander) remove the vertical equality sign on the far right of the diagram (otherwise it doesn't commute).

-p. 487, l. -10 ff (noticed by B. Selander) replace ' $H^0(k_{\mathfrak{p}}, A')$ ' by ' $C^0(k_{\mathfrak{p}}, A')$ ' and ' $H^0(G_S, I_S(A))$ ' by ' $C^0(G_S, I_S(A))$ '. In line -7 replace '0-cochain' by '0-cocycle'. After this sentence add the sentence 'Moreover, because the 2-cochain  $z \in C^2(G_S, \mathcal{O}_S^{\times})$  maps to zero in  $C^2(G_S, C_S)$ , the 2-cocycle  $(y'_{\mathfrak{p}} \cup x_{\mathfrak{p}} - z_{\mathfrak{p}}) \in Z^2(G_S, I_S)$  maps to  $y' \cup x \in Z^2(G_S, C_S)$ . Then remove the commutative diagram and the final phrase of the proof starting with 'because the image ...'.

-p. 620, l. -9 (noticed by anonymous) replace  $Cl_0$  by  $Cl_S$ . Subsequently, the next three lines and the footnote become obsolete. -p. 730, l. 5, (noticed by C. Lane) the word 'obstruction' is poorly chosen here.

- -p. 731, l. -3, (noticed by C. Lane) add 'and ex. 7 of §5.2'.
- **-p. 782, l. 8 and 10** replace  $e_i Cl(k)$  by  $e_i Cl(k)(p)$ .
- -p. 782, l. 8 replace ' $L(1, \omega^i)$ ' by ' $L(0, \omega^i)$ '.
- -p. 782, l. 9 replace  $(L(1, \omega^i))$  by  $(L(0, \omega^{-i}))$ .

## Errata for the printed version (Corrected Second Printing 2013)

-p. X, l. -18 and -16 (noticed by T. Wunder) replace '§4' by '§5' and '§5' by '§6'.

-p. 8, l. 18–21 (noticed by Siyan "Daniel" Li) replace 'If  $A = A^U$  for some open subgroup  $U \subseteq G$ , then Hom(A, B) is a discrete G-module. This is the case, for example,' by 'Hom(A, B) is a discrete G-module'.

- -p. 18, l. 5 The section s must be chosen such that s(1) = 1. This is possible after a translation.
- -p. 18, l. 13 remove 'and A is closed in  $\hat{G}$ '.
- -p. 23, l. -1 (noticed by D. Vogel) replace '§8' by '§9'.
- -p. 32, footnote (noticed by M. Lüdtke) replace '§7' by '§8'.
- -p. 39, l. 4 (noticed by D. Harari) replace 'C' by 'C''.

-p. 59, Ex. 8 (noticed by C. Lane) The formulation of the exercise is messed up.

-p. 62, l. 3 (noticed by S. Panda) Replace ', which' by '. If H is of finite index in G, it' and remove 'H is of finite index in G and' in line 5.

-p. 73, Exercises 2 and 3 (noticed by M. Lüdtke) In both exercises the subgroup H should be normal.

**-p. 89** (noticed by A. Holschbach) add before Lemma (1.9.8): 'For an abelian profinite group A and a prime number p, we denote by A(p) the (unique) p-Sylow subgroup of A.'

-p. 144, l. 14 replace  ${}^{\prime}_{\ell^m}H^n_{cts}(G,T)$  by  ${}^{\prime}_{\ell^m}H^{n+1}_{cts}(G,T)$ .

-p. 181, Exercise 5 (noticed by anonymous) replace the assumption  $cd_pG/H \neq 0$  by  $cd_pH \neq 0$ .

-p. 199, l. 9 (misleading argument) replace 'i:  $X \to F$ . It satisfies condition (1) of (3.5.14), since A is finite' by 'i:  $X \to F$ , which satisfies condition (1) of (3.5.14).'

- -p. 209, Ex. 2 (noticed by C. Lane) Add the assumption that the curve in question has genus  $\geq 1$ .
- -p. 210, l. 22 (noticed by O. Thomas) replace 'for A' by 'for finite A'.
- **-p. 218, l. 12–15** (noticed by J. Minac) replace '*H*' by '*U*' (three times).
- -p. 219, l. -4 (noticed by K. Koziol) replace 'a Poincaré group' by 'a pro-p Poincaré group'.
- -p. 228, l. 2 (noticed by A. Holschbach) replace 'X §8' by 'X §10'.
- -p. 228, l. -14 (noticed by A. Holschbach) replace 'ker $(j|_G)$ ' by 'ker $(j|_{A^G})$ '.
- -p. 239, l. 4 replace  $2^{f}$ , by  $2^{f-1}$ .
- **-p. 239, l. 5** replace '2<sup>f</sup>' by '2'.
- -p. 239, l. 17 replace 'alternating' by 'anti-symmetric'.
- **-p. 266, l. 12** (noticed by anonymous) replace 'of  $G_t \subseteq$ ' by 'of  $G_t \times G_t \subseteq$ '.
- **-p. 271, l. 12** (noticed by anonymous) replace (1, ..., n) by (1, ..., h).

-p. 290, l. 13 (noticed by L. Sauer) replace 'deg( $\bar{v}$ ) = 0' by 'the constant term of v is a unit in  $\mathcal{O}$ '.

-p. 310, l. -11 ff (noticed by O. Thomas) replace 'for  $D_{r-1}(M^{\vee})$ . Therefore  $D_{r-1}(M^{\vee}) \otimes \mathbb{Q}/\mathbb{Z} = 0$  and' by 'for  $D_{r-1}(M^{\vee})$  and  $D_r(M^{\vee})$ . Therefore  $D_r(M^{\vee}) \otimes \mathbb{Q}/\mathbb{Z} = 0$  and'.

-p. 326, l. -12 (noticed by C. Lane) replace  ${}^{*}N^{ab}_{\mathscr{H}}/G$  by  ${}^{*}N^{ab}_{\mathscr{H}}/\operatorname{Rad}_{G}$ .

-p. 340, l. -7 (noticed by Siyan "Daniel" Li) the explicit formula for the map is wrong and has to be replaced by

$$(a_0, \dots, a_{n-1}) \longmapsto \tilde{a}_0^{p^{n-1}} + \tilde{a}_1^{p^{n-2}} p + \dots + \tilde{a}_{n-1} p^{n-1} \mod p^n,$$

where  $\tilde{a}_i$  is some lift of  $a_i$  to  $\mathbb{Z}$ .

-p. 393, (7.3.3) Lemma. The lemma is correct but not sufficient for the application in the proof of (7.3.2). It should be replaced by

(7.3.3) Lemma.

(i) Let A be a finite G-module. Then

 $[\ell A] = [A_\ell].$ 

(ii) Let V be a G-module such that  $V_{\ell}$  and  $_{\ell}V$  are finite. If  $W \subseteq V$  is a submodule of finite index, then

$$[V_{\ell}] - [_{\ell}V] = [W_{\ell}] - [_{\ell}W].$$

**Proof:** We prove (ii) first. Using a Jordan-Hölder series, we may assume that V/W is a finite simple G-module. In particular,  $\ell(V/W) \cong (V/W)_{\ell}$ . Consider the diagram

The snake lemma gives the exact sequence

 $0 \to_{\ell} W \to_{\ell} V \to_{\ell} (V/W) \to W_{\ell} \to V_{\ell} \to (V/W)_{\ell} \to 0,$ 

and hence the result. Assertion (i) follows by applying (ii) to V = A, W = 0.

-p. 408 , l. 3 (noticed by C. Lane) replace ' $T_k$ ' by ' $G_k$ '.

-p. 450, l. -11 (noticed by M. Leonhardt) Replace  $H^0(G, D_K) = D_k$  by  $H^0(G, D_K) \cong D_k \oplus (\mathbb{Z}/2\mathbb{Z})^m$ .

-p. 450, l.-10 and l.-7 (noticed by M. Leonhardt) Replace  $S_{\mathbb{C}}$  by  $S_{\mathbb{C}}(K)$ .

-p. 451, l. -9 (noticed by M. Leonhardt) Replace the last paragraph of the proof of (8.2.6) by: Finally note that  $N_{K|k}D_K = D_k$  by (8.2.1)(iv) and that  $D_k$  is divisible by (8.2.1)(vi). Hence the calculation of  $H^0(G, D_K)$  follows from that of  $\hat{H}^0(G, D_K)$ .

-p. 460, l. 6 (noticed by A. Holschbach) replace 'onto' by 'into'.

-p. 466, Ex. 1 (noticed by C. Lane) The statement of the exercise is false. It becomes correct after replacing  $\mathcal{O}_S^{\times}$  by  $k_S^{\times}$ . (However, then it is a special case of (6.3.4).)

**-p. 484, bottom** (noticed by B. Selander) remove the vertical equality sign on the far right of the diagram (otherwise it doesn't commute).

-p. 487, l. -10 ff (noticed by B. Selander) replace  ${}^{\prime}H^{0}(k_{\mathfrak{p}}, A')$ ' by  ${}^{\prime}C^{0}(k_{\mathfrak{p}}, A')$ ' and  ${}^{\prime}H^{0}(G_{S}, I_{S}(A))$ ' by  ${}^{\prime}C^{0}(G_{S}, I_{S}(A))$ '. In line -7 replace '0-cochain' by '0-cocycle'. After this sentence add the sentence 'Moreover, because the 2-cochain  $z \in C^{2}(G_{S}, \mathcal{O}_{S}^{\times})$  maps to zero in  $C^{2}(G_{S}, C_{S})$ , the 2-cocycle  $(y'_{\mathfrak{p}} \cup x_{\mathfrak{p}} - z_{\mathfrak{p}}) \in Z^{2}(G_{S}, I_{S})$  maps to  $y' \cup x \in Z^{2}(G_{S}, C_{S})$ . Then remove the commutative diagram and the final phrase of the proof starting with 'because the image ...'.

-p. 498, I. -1 replace 
$$\lim \lambda'_T = \pi_T (\lim \lambda'_S)$$
 by  $\lim \lambda'_T = \pi_T (\lim (\lambda'_S \circ inf))$ 

-p. 507, l. -2 (noticed by A. Holschbach) replace 'finite subextension' by 'finite, totally imaginary subextension'

-p. 532, l. 5 (noticed by A. Holschbach) replace (k, S, m)' by (k, m, S)'.

-p. 554, 1.17 (noticed by A. Holschbach) The word 'realize' might be misleading. To be more precise, one should replace the first sentence of the proof of (9.4.3) by the following: 'We have to show that for every finite Galois extension  $K_{\mathfrak{p}}|k_{\mathfrak{p}}$  with  $G(K_{\mathfrak{p}}|k_{\mathfrak{p}}) \in \mathfrak{c}$  there exists a global Galois extension L|k unramified outside S with  $G(L|k) \in \mathfrak{c}$  such that  $K_{\mathfrak{p}} \subset L_{\mathfrak{p}}$ .'

-p. 569, l. -7 (noticed by M. Jarden) For  $i \ge 2$ ,  $A_i$  is not a  $\Gamma$ -module. However,  $A_i$  is a  $G/H_{i-1}$ -module and, after refining the filtration, we may assume without loss of generality that it is simple.

-p. 600, l. -15 (noticed by T. Wunder) replace '1' by '2'.

-p. 620, l. -9 (noticed by anonymous) replace  $Cl_0$  by  $Cl_S$ . Subsequently, the next three lines and the footnote become obsolete.

**-p. 621, l. -12** (noticed by T. Wunder) replace 'k' by 'K' (twice).

-p. 622, l. 15 (noticed by T. Wunder) replace  $G_T$  by  $G_T(\mathfrak{c})$ .

-p. 623, l. -8 (noticed by T. Wunder) replace 'chapter XII' by '§11'.

-p. 630, l. 7 (noticed by D. Neugber) replace 'For S' by 'For finite S'.

-p. 637, l. 7 (noticed by T. Wunder) replace 'G' by ' $G_K$ '.

-p. 649, l. 10ff (noticed by D. Neugher) replace 'Comparing two copies of the upper sequence of (10.3.13) (for T and S) we therefore obtain the commutative exact diagram (writing  $E_{k'}$  for  $\mathcal{O}_{k'}^{\times}$  and  $G_T$  for  $G_T(k')$ )' by 'Passing to the inverse limit over the upper exact sequences of (10.3.13) for  $T = \emptyset$  and all finite subsets of S(k') containing  $S_p \cup S_{\infty}$ , and doing the same for T instead of S, we obtain the commutative exact diagram (writing  $E_{k'}$  for  $\mathcal{O}_{k'}^{\times}$ ,  $G_S$  for  $G_S(k')$  and  $G_T$  for  $G_T(k')$ )'.

-p. 650 (10.5.5) Corollary. (noticed by M. Witte) replace 'is an isomorphism' by 'is surjective'. In the proof replace ' $E_2^{2,0} = E_{\infty}^{2,0}$ ' by  $E_2^{2,0} \rightarrow E_{\infty}^{2,0}$ '.

-p. 652 ff. (noticed by O. Thomas) According to our convention in section 10.5, the number field k in (10.5.8)–(10.5.11) should be a *finite* number field (otherwise we cannot speak about density)

- -p. 653, l. -4 (noticed by O. Thomas) replace  $H^i(G(k_p(p)|k'_p))$  by  $H^i(G(k_p(p)|k'_p))$ .
- -p. 679, l. -12 (noticed by O. Thomas) replace '(10.5.1)(i)' by '(10.5.1)'.
- -p. 683, l. -4 (noticed by O. Thomas) replace  $I_S(k)$  by  $I_S(k)/p$ .
- -p. 695, l. 7 (noticed by T. Wunder) replace  $k_S/K$  by  $k_S|K$ .
- -p. 689, l. -5,-2, p. 690 l. 1 (noticed by M. Witte) replace ' $\text{III}^2(k_S, S_0, -)$ ' by ' $\text{III}^2(k_S, S \smallsetminus S_0, -)$ '.
- -p. 701, l. -10, (noticed by O. Thomas) replace Cl(k) by  $Cl_T(k)$ .
- -p. 730, l. 5, (noticed by C. Lane) the word 'obstruction' is poorly chosen here.
- -p. 731, l. -3, (noticed by C. Lane) add 'and ex. 7 of §5.2'.
- -p. 781, l. 11, p. 784, l. 16 (noticed by H. Johnston) replace '[61]' by '[64]'.
- **-p. 782, l. 8 and 10** replace  $e_i Cl(k)$  by  $e_i Cl(k)(p)$ .
- -**p. 782, l. 8** replace  $L(1, \omega^i)$  by  $L(0, \omega^i)$ .
- -p. 782, l. 9 replace  $L(1, \omega^i)$  by  $L(0, \omega^{-i})$ .
- -p. 789, l. 16 (noticed by M. van Frankenhuijsen) replace 'q=' by 'q-1='.
- -p. 796, l. 16 remove '(i.e. p splits completely in  $k_2|\mathbb{Q}$ ),'
- -p. 815, [181] (noticed by T. Keller) replace 'construction' by 'corestriction'.