Errata for:

Cohomology of Number Fields. Online Edition 2.3, May 2020

by J. Neukirch, A. Schmidt, K. Wingberg

This file lists known mistakes. If you find a mistake not listed below or have a comment, please send me an e-mail: schmidt@mathi.uni-heidelberg.de.

-p. 18, l. 5 The section s must be chosen such that $s(1) = 1$. This is possible after a translation.
-p. 18, l. 13 remove 'and A is closed in $\hat{G}$'.
-p. 21, l. -6 (noticed by D. Vogel) replace '§8' by '§9'.
-p. 23, l. -1 (noticed by D. Vogel) replace '§8' by '§9'.
-p. 144, l. 14 replace '${}_{\ell^m}H^n_{cts}(G,T)$' by '${}_{\ell^m}H^{n+1}_{cts}(G,T)$'.
-p. 219, l. -4 (noticed by K. Koziol) replace 'a Poincaré group' by 'a pro-p Poincaré group'.
-p. 466, Ex. 1 (noticed by C. Lane) The statement of the exercise is false. It becomes correct after replacing '$\mathcal{O}_S^\times$' by '$k_S^\times$'. (However, then it is a special case of (6.3.4).)
-p. 484, bottom (noticed by B. Selander) remove the vertical equality sign on the far right of the diagram (otherwise it doesn't commute).
-p. 487, l. -10 ff (noticed by B. Selander) replace '$H^0(k_\mathfrak{p}, A')$' by '$C^0(k_\mathfrak{p}, A')$' and '$H^0(G_S, I_S(A))$' by '$C^0(G_S, I_S(A))$'. In line -7 replace '0-cochain' by '0-cocycle'. After this sentence add the sentence 'Moreover, because the 2-cochain $z \in C^2(G_S, \mathcal{O}_S^\times)$ maps to zero in $C^2(G_S, C_S)$, the 2-cocycle $(y'_\mathfrak{p} \cup x_\mathfrak{p} - z_\mathfrak{p}) \in Z^2(G_S, I_S)$ maps to $y' \cup x \in Z^2(G_S, C_S)$. Then remove the commutative diagram and the final phrase of the proof starting with 'because the image …'.
-p. 620, l. -9 (noticed by anonymous) replace Cl_0 by Cl_S. Subsequently, the next three lines and the footnote become obsolete.
-p. 782, l. 8 and 10 replace '$e_i Cl(k)$' by '$e_i Cl(k)(p)$'.
-p. 782, l. 8 replace '$L(1,\omega^i)$' by '$L(0,\omega^i)$'.
-p. 782, l. 9 replace '$L(1,\omega^i)$' by '$L(0,\omega^{-i})$'.

Errata for the printed version (Corrected Second Printing 2013)

-p. X, l. -18 and -16 (noticed by T. Wunder) replace '§4' by '§5' and '§5' by '§6'.
-p. 8, l. 18–21 (noticed by Siyan "Daniel" Li) replace 'If $A = A^U$ for some open subgroup $U \subseteq G$, then $\mathrm{Hom}(A,B)$ is a discrete G-module. This is the case, for example,' by '$\mathrm{Hom}(A,B)$ is a discrete G-module'.
-p. 18, l. 5 The section s must be chosen such that $s(1) = 1$. This is possible after a translation.
-p. 18, l. 13 remove 'and A is closed in $\hat{G}$'.
-p. 23, l. -1 (noticed by D. Vogel) replace '§8' by '§9'.
-p. 32, footnote (noticed by M. Lüdtke) replace '§7' by '§8'.
-p. 39, l. 4 (noticed by D. Harari) replace 'C' by 'C''.
-p. 62, l. 3 (noticed by S. Panda) Replace ', which' by '. If H is of finite index in G, it' and remove 'H is of finite index in G and' in line 5.
-p. 73, Exercises 2 and 3 (noticed by M. Lüdtke) In both exercises the subgroup H should be normal.
-p. 89 (noticed by A. Holschbach) add before Lemma (1.9.8): 'For an abelian profinite group A and a prime number p, we denote by $A(p)$ the (unique) p-Sylow subgroup of A.'
-p. 144, l. 14 replace '${}_{\ell^m}H^n_{cts}(G,T)$' by '${}_{\ell^m}H^{n+1}_{cts}(G,T)$'.
-p. 181, Exercise 5 (noticed by anonymous) replace the assumption '$cd_p G/H \neq 0$' by '$cd_p H \neq 0$'.
-p. 199, l. 9 (misleading argument) replace '$i\colon\ X \to F$. It satisfies condition (1) of (3.5.14), since A is finite' by '$i\colon\ X \to F$, which satisfies condition (1) of (3.5.14).'
-p. 210, l. 22 (noticed by O. Thomas) replace 'for A' by 'for finite A'.
-p. 218, l. 12–15 (noticed by J. Minac) replace 'H' by 'U' (three times).
-p. 219, l. -4 (noticed by K. Koziol) replace 'a Poincaré group' by 'a pro-p Poincaré group'.
-p. 228, l. 2 (noticed by A. Holschbach) replace 'X §8' by 'X §10'.
-p. 228, l. -14 (noticed by A. Holschbach) replace '$\ker(j|_G)$' by '$\ker(j|_{A^G})$'.
-p. 239, l. 4 replace '2^f' by '2^{f-1}'.
-p. 239, l. 5 replace '2^f' by '2'.
-p. 239, l. 17 replace 'alternating' by 'anti-symmetric'.
-p. 266, l. 12 (noticed by anonymous) replace 'of $G_t \subseteq$' by 'of $G_t \times G_t \subseteq$'.
-p. 271, l. 12 (noticed by anonymous) replace '$1,\dots,n$' by '$1,\dots,h$'.
-p. 290, l. 13 (noticed by L. Sauer) replace '$\deg(\bar{v}) = 0$' by 'the constant term of v is a unit in $\mathcal{O}$'.
-p. 310, l. -11 ff (noticed by O. Thomas) replace 'for $D_{r-1}(M^\vee)$. Therefore $D_{r-1}(M^\vee) \otimes \mathbb{Q}/\mathbb{Z} = 0$ and' by 'for $D_{r-1}(M^\vee)$ and $D_r(M^\vee)$. Therefore $D_r(M^\vee) \otimes \mathbb{Q}/\mathbb{Z} = 0$ and'.
-p. 340, l. -7 (noticed by Siyan "Daniel" Li) the explicit formula for the map is wrong and has to be replaced by

$$(a_0,\dots,a_{n-1}) \longmapsto \tilde{a}_0^{p^{n-1}} + \tilde{a}_1^{p^{n-2}} p + \cdots + \tilde{a}_{n-1} p^{n-1} \mod p^n,$$

where $\tilde{a}_i$ is some lift of a_i to $\mathbb{Z}$.

-p. 393, (7.3.3) Lemma. The lemma is correct but not sufficient for the application in the proof of (7.3.2). It should be replaced by

(7.3.3) Lemma.

(i) *Let A be a finite G-module. Then*

$$[{}_\ell A] = [A_\ell].$$

(ii) *Let V be a G-module such that V_ℓ and ${}_\ell V$ are finite. If $W \subseteq V$ is a submodule of finite index, then*

$$[V_\ell] - [{}_\ell V] = [W_\ell] - [{}_\ell W].$$

Proof: We prove (ii) first. Using a Jordan-Hölder series, we may assume that V/W is a finite simple G-module. In particular, ${}_\ell(V/W) \cong (V/W)_\ell$. Consider the diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & W & \longrightarrow & V & \longrightarrow & V/W & \longrightarrow & 0 \\ & & \downarrow \ell & & \downarrow \ell & & \downarrow \ell & & \\ 0 & \longrightarrow & W & \longrightarrow & V & \longrightarrow & V/W & \longrightarrow & 0. \end{array}$$

The snake lemma gives the exact sequence

$$0 \to {}_\ell W \to {}_\ell V \to {}_\ell(V/W) \to W_\ell \to V_\ell \to (V/W)_\ell \to 0,$$

and hence the result. Assertion (i) follows by applying (ii) to $V = A$, $W = 0$. □

-p. 450, l. -11 (noticed by M. Leonhardt) Replace '$H^0(G, D_K) = D_k$' by '$H^0(G, D_K) \cong D_k \oplus (\mathbb{Z}/2\mathbb{Z})^m$'.

-p. 450, l.-10 and l.-7 (noticed by M. Leonhardt) Replace '$S_\mathbb{C}$' by '$S_\mathbb{C}(K)$'.

-p. 451, l. -9 (noticed by M. Leonhardt) Replace the last paragraph of the proof of (8.2.6) by: Finally note that $N_{K|k} D_K = D_k$ by (8.2.1)(iv) and that D_k is divisible by (8.2.1)(vi). Hence the calculation of $H^0(G, D_K)$ follows from that of $\hat{H}^0(G, D_K)$.

-p. 460, l. 6 (noticed by A. Holschbach) replace 'onto' by 'into'.

-p. 466, Ex. 1 (noticed by C. Lane) The statement of the exercise is false. It becomes correct after replacing '$\mathcal{O}_S^\times$' by '$k_S^\times$'. (However, then it is a special case of (6.3.4).)

-p. 484, bottom (noticed by B. Selander) remove the vertical equality sign on the far right of the diagram (otherwise it doesn't commute).

-p. 487, l. -10 ff (noticed by B. Selander) replace '$H^0(k_\mathfrak{p}, A')$' by '$C^0(k_\mathfrak{p}, A')$' and '$H^0(G_S, I_S(A))$' by '$C^0(G_S, I_S(A))$'. In line -7 replace '0-cochain' by '0-cocycle'. After this sentence add the sentence 'Moreover, because the 2-cochain $z \in C^2(G_S, \mathcal{O}_S^\times)$ maps to zero in $C^2(G_S, C_S)$, the 2-cocycle $(y'_\mathfrak{p} \cup x_\mathfrak{p} - z_\mathfrak{p}) \in Z^2(G_S, I_S)$ maps to $y' \cup x \in Z^2(G_S, C_S)$. Then remove the commutative diagram and the final phrase of the proof starting with 'because the image ...'.

-p. 498, l. -1 replace '$\mathrm{im}\ \lambda'_T = \pi_T(\mathrm{im}\ \lambda'_S)$' by '$\mathrm{im}\ \lambda'_T = \pi_T(\mathrm{im}(\lambda'_S \circ \mathit{inf}))$'.

-p. 507, l. -2 (noticed by A. Holschbach) replace 'finite subextension' by 'finite, totally imaginary subextension'

-p. 532, l. 5 (noticed by A. Holschbach) replace '(k, S, m)' by '(k, m, S)'.

-p. 554, l.17 (noticed by A. Holschbach) The word 'realize' might be misleading. To be more precise, one should replace the first sentence of the proof of (9.4.3) by the following: 'We have to show that for every finite Galois extension $K_\mathfrak{p}|k_\mathfrak{p}$ with $G(K_\mathfrak{p}|k_\mathfrak{p}) \in \mathfrak{c}$ there exists a global Galois extension $L|k$ unramified outside S with $G(L|k) \in \mathfrak{c}$ such that $K_\mathfrak{p} \subset L_\mathfrak{p}$.'

-p. 569, l. -7 (noticed by M. Jarden) For $i \geq 2$, A_i is not a Γ-module. However, A_i is a G/H_{i-1}-module and, after refining the filtration, we may assume without loss of generality that it is simple.

-p. 600, l. -15 (noticed by T. Wunder) replace '1' by '2'.

-p. 620, l. -9 (noticed by anonymous) replace Cl_0 by Cl_S. Subsequently, the next three lines and the footnote become obsolete.

-p. 621, l. -12 (noticed by T. Wunder) replace 'k' by 'K' (twice).

-p. 622, l. 15 (noticed by T. Wunder) replace 'G_T' by '$G_T(\mathfrak{c})$'.

-p. 623, l. -8 (noticed by T. Wunder) replace 'chapter XII' by '§11'.

-p. 630, l. 7 (noticed by D. Neugber) replace 'For S' by 'For finite S'.

-p. 637, l. 7 (noticed by T. Wunder) replace 'G' by 'G_K'.

-p. 649, l. 10ff (noticed by D. Neugber) replace 'Comparing two copies of the upper sequence of (10.3.13) (for T and S) we therefore obtain the commutative exact diagram (writing $E_{k'}$ for $\mathcal{O}_{k'}^\times$ and G_T for $G_T(k')$)' by 'Passing to the inverse limit over the upper exact sequences of (10.3.13) for $T = \varnothing$ and all finite subsets of $S(k')$ containing $S_p \cup S_\infty$, and doing the same for T instead of S, we obtain the commutative exact diagram (writing $E_{k'}$ for $\mathcal{O}_{k'}^\times$, G_S for $G_S(k')$ and G_T for $G_T(k')$)'.

-p. 650 (10.5.5) Corollary. (noticed by M. Witte) replace 'is an isomorphism' by 'is surjective'. In the proof replace '$E_2^{2,0} = E_\infty^{2,0}$' by $E_2^{2,0} \twoheadrightarrow E_\infty^{2,0}$''.

-p. 652 ff. (noticed by O. Thomas) According to our convention in section 10.5, the number field k in (10.5.8)–(10.5.11) should be a *finite* number field (otherwise we cannot speak about density)

-p. 653, l. -4 (noticed by O. Thomas) replace '$H^i(G(k_p(p)|k'_\mathfrak{p}))$' by '$H^i(G(k_\mathfrak{p}(p)|k'_\mathfrak{p}))$'.

-p. 679, l. -12 (noticed by O. Thomas) replace '(10.5.1)(i)' by '(10.5.1)'.

-p. 683, l. -4 (noticed by O. Thomas) replace '$I_S(k)$' by '$I_S(k)/p$'.

-p. 695, l. 7 (noticed by T. Wunder) replace 'k_S/K' by '$k_S|K$'.

-p. 689, l. -5,-2, p. 690 l. 1 (noticed by M. Witte) replace '$Ш^2(k_S, S_0, -)$' by '$Ш^2(k_S, S \smallsetminus S_0, -)$'.

-p. 701, l. -10, (noticed by O. Thomas) replace '$Cl(k)$' by '$Cl_T(k)$'.

-p. 781, l. 11, p. 784, l. 16 (noticed by H. Johnston) replace '[61]' by '[64]'.

-p. 782, l. 8 and 10 replace '$e_i\, Cl(k)$' by '$e_i\, Cl(k)(p)$'.

-p. 782, l. 8 replace '$L(1, \omega^i)$' by '$L(0, \omega^i)$'.
-p. 782, l. 9 replace '$L(1, \omega^i)$' by '$L(0, \omega^{-i})$'.
-p. 789, l. 16 (noticed by M. van Frankenhuijsen) replace 'q=' by 'q-1='.
-p. 796, l. 16 remove '(i.e. p splits completely in $k_2|\mathbb{Q}$),'
-p. 815, [181] (noticed by T. Keller) replace 'construction' by 'corestriction'.

last update: March 5, 2024 by **Alexander Schmidt**