

Program of the workshop

$G_{\mathbb{Q}_p}$ as a geometric fundamental group

February 23, 2015

Let us recall the formal strategy for the proof of the main result in the case $E = \mathbb{Q}_p$. We write C for the completion of an algebraic closure of \mathbb{Q}_p . One shows the existence of an perfectoid space U^1 over C such that its tilt U^b is also the tilt of $Y^{ad} \otimes \mathbb{Q}_p^{cyc}$, where \mathbb{Q}_p^{cyc} denotes the completion of the $\mathbb{Q}_p(\mu_{p^\infty})$ while Y^{ad} denotes the adic space such that $X^{ad} = Y^{ad}/\varphi^{\mathbb{Z}}$ gives the adic space attached to the (schematic) Fargues-Fontaine curve X . By Scholze's tilting equivalence this induces equivalences of the corresponding categories of finite étale covers

$$(U)_{f\acute{e}t} \cong (U^b)_{f\acute{e}t} \cong (Y_{C^b, \mathbb{Q}_p}^{ad} \otimes \mathbb{Q}_p^{cyc})_{f\acute{e}t}$$

which in turn induces equivalences

$$(U/p^{\mathbb{Z}})_{f\acute{e}t} \cong (U^b/p^{\mathbb{Z}})_{f\acute{e}t} \cong (X_{C^b, \mathbb{Q}_p}^{ad} \otimes \mathbb{Q}_p^{cyc})_{f\acute{e}t} \cong (\mathbb{Q}_p^{cyc})_{f\acute{e}t}$$

by dividing out the compatible action of p and φ , respectively. The last equivalence follows by the fact that X (and hence X^{ad} by a GAGA principle) is geometrically simply connected. This equivalence turns out to be compatible with a common $\mathbb{Z}_p^\times = G(\mathbb{Q}_p(\mu_{p^\infty})/\mathbb{Q}_p)$ -action and hence - using the theory of field of norms - induces an equivalence

$$(Z)_{f\acute{e}t} \cong (\mathbb{Q}_p)_{f\acute{e}t}$$

where $Z = U/\mathbb{Q}_p^\times$ is the desired object (which only exists as a sheaf on the category of perfectoid spaces over C endowed with the pro-étale topology) which is isomorphic to the diamond $(\tilde{D}_C^*)^\diamond/\mathbb{Q}_p^\times$ in the language of Scholze, see [SchLec, §17].

The strategy pursued in [Wei] differs slightly from the above in that the Fargues-Fontaine curve in characteristic 0 can be completely avoided by using the following equivalences

$$(U^b/p^{\mathbb{Z}})_{f\acute{e}t} \cong (X_{C^b, L(\mathbb{Q}_p)}^{ad} \otimes \mathbb{Q}_p^{cyc, b})_{f\acute{e}t} \cong (\mathbb{Q}_p^{cyc, b})_{f\acute{e}t} \cong (\mathbb{Q}_p^{cyc})_{f\acute{e}t}$$

and the classification of vector bundles on $X_{C^b, L(\mathbb{Q}_p)}^{ad}$ by Hartl/Pink. Here $L(\mathbb{Q}_p)$ denotes the imperfect field of norms attached to the extension $\mathbb{Q}_p(\mu_{p^\infty})/\mathbb{Q}_p$ and the last equivalence is again the tilting equivalence.

¹ U is the 'perfectoid cover' $\varprojlim_{x \mapsto (1+x)^{p-1}} D_C^*$ of the punctured open unit disc D_C centered at 0, the latter considered as rigid analytic space over C , as in the introduction of [Wei].

We aim to have 23 talks, each 60 minutes, from Monday to Thursday 5 talks per day, on Friday only 3 talks. Thematically they are grouped according to the following topics:

AS I-III Adic spaces WEDHORN

explain basics of the theory of adic spaces, cover at least [Sch1, §2], compare also [SchLec, 2-5]; explain étale morphisms of adic spaces, discuss the rigid open disc as adic generic fibre [Wei, 3.1.4], see also [SchLec, 5.4].

PF Perfectoid fields WULKAU

explain [Sch1, §3,4], compare also [SchLec, 6].

PA I,II Perfectoid algebras GROSSE-KLÖNNE AND SCHMIDT

explain [Sch1, §5], compare also [SchLec, 6].

PS I,II Perfectoid Spaces BORNMAN AND FELDMANN

explain [Sch1, §6], compare also [SchLec, 7].

PS III Perfectoid Spaces SCHNEIDER

discuss perfectoid generic fibres [Wei, §3.2] as well as the example $\varprojlim_{x \mapsto (1+x)^p - 1} D_C^*$ [SchLec, 11.1.4] and its tilt.

TE Tilting equivalence for the étale sites CHEN

explain [Sch1, §7.1-7.12], compare also [SchLec, 7.3-7.5].

FF I-IV The Fargues-Fontaine curve $X_{F,E}$ and its adic version $X_{F,E}^{ad}$ HEIDELBERG GROUP

assume always F algebraically closed. Explain [Durham, 1-5], [F, 1-2], compare also [SchLec, 13.5].

VB I-III Vector bundles on $X_{F,E}$ HEIDELBERG GROUP

assume always F algebraically closed. Explain [Durham, 6.1-6.3], [F, 3.1,3.2], compare also [FF, §2-3,§12].

GAGA GAGA for $X_{F,E}$ to $X_{F,E}^{ad}$ VENJAKOB

explain [Durham, 7.5,7.6], [F, 3.3], compare also [SchLec, Thm. 13.5.6]; mention that alternatively one can classify the vector bundles on $X_{C^b, L(\mathbb{Q}_p)}^{ad}$ directly following Hartl/Pink, see [Wei, §3.7].

SC $X_{F,E}$ and $X_{F,E}^{ad}$ are geometrically simply connected GÖRTZ

explain [FF, §18], [Wei, Lem. 3.8.2], see also the technique of proof of Proposition 3.8.3 in (loc. cit.) as well as [SchLec, Thm. 13.5.7]. Also explain, for an adic space Z over E , the equivalence of the following conditions:

- $Z \otimes_E \hat{E}$ is simply connected,
- $Z_{f\acute{e}t} \cong \text{spec}(E)_{f\acute{e}t}$.

E I,II Etale fundamental group and pro-étale topology KOHLHAASE AND WITTE

recall the definition of the fundamental group, e.g. from [Milne, I §5], explain [Sch2, §3,§4.3-4.8], compare also [SchLec, 7.5,8.2].

MR I,II **Main result** MIHATSCH AND HELLMANN

determine first the tilts of Y^{ad} and X^{ad} according to [Wei, §3.4,3.5], then explain [Wei, §4], compare also [SchLec, 17.3.6,§18].

The schedule is planned as follows:

Mo	Tu	We	Th	Fr
AS I	AS III	PS II	VB II	SC
PF	PA II	FF IV	E I	MR I
AS II	FF II	PS III	VB III	MR II
PA I	PS I	VB I	GAGA	
FF I	FF III	TE	E II	

References

- [F] Fargues L.: *Quelques resultats et conjectures concernant la courbes*. <http://webusers.imj-prg.fr/~laurent.fargues/AuDela.pdf>
- [Durham] L. Fargues and J.-M. Fontaine.: *Vector bundles on curves and p-adic hodge theory*. A paraitre aux Proceedings du Symposium EPSRC *Automorphic forms and Galois representations* ayant eu lieu à Durham en juillet 2011.
- [FF] Fargues L. and Fontaine J.-M.: *Courbes et fibrés vectoriels en théorie de Hodge p-adique*. <http://webusers.imj-prg.fr/~laurent.fargues/Prepublications.html>
- [Milne] Milne, J. S.: *Étale cohomology*. Princeton Mathematical Series, 33. Princeton University Press, Princeton, N.J., 1980. xiii+323 pp. ISBN: 0-691-08238-3
- [Sch1] Scholze P.: *Perfectoid spaces*. Publ. Math. IHES 116 (2012), 245–313
- [Sch2] Scholze P.: *p-adic Hodge theory for rigid-analytic varieties*. Forum Math. Pi 1 (2013)
- [SchLec] *Peter Scholze’s lectures on p-adic geometry, Fall 2014*. Notes by J. Weinstein, version from 19.12.2015, <http://math.berkeley.edu/~jared/Math274/ScholzeLectures.pdf>
- [Wei] Weinstein J.: *$Gal(\overline{\mathbb{Q}_p}/\mathbb{Q}_p)$ as a geometric fundamental group*. arXiv:1404.7192v1