1. Case: $\varrho = \mathrm{Ind}_{\mathcal{G}}^{G}\chi$, i. e. $\breve{\varrho}_{|_G} = \bar{\chi} \oplus \bar{\chi}^c$

where $\chi^c(g) := \chi(cgc^{-1})$ with
$c \in \mathcal{G}$ complex conjugation

$$\frac{\mathcal{L}(\breve{\varrho})}{\mathcal{L}_E(\breve{\varrho})} = \frac{\mathcal{L}_{\bar{\psi}}(\bar{\chi}) \cdot \mathcal{L}_{\bar{\psi}}(\bar{\chi}^c)}{\mathcal{L}_E(\breve{\varrho})} = \frac{\frac{\Omega_p^2}{\Omega^2}}{\Omega_+^{-1}\Omega_-^{-1}} \; \frac{G(\bar{\chi})G(\bar{\chi}^c)}{u^{-\mathfrak{f}_p(\varrho)}\, e_p(\varrho)}$$

$$\cdot \; \frac{P_{\mathfrak{p}}(\bar{\chi}, u^{-1}) P_{\bar{\mathfrak{p}}}(\chi, u^{-1}) P_{\mathfrak{p}}(\bar{\chi}^c, u^{-1}) P_{\bar{\mathfrak{p}}}(\chi^c, u^{-1})}{P_p(\breve{\varrho}, u^{-1}) P_p(\varrho, w^{-1})^{-1} P_p(E, \rho, \frac{1}{p})} \; \frac{L(\bar{\psi}\chi, 1)\, L(\bar{\psi}\chi, 1)}{L(E, \varrho, 1)}$$

$$= \frac{\Omega_p^2 \;\; \Omega_+\Omega_-}{\Omega \quad \Omega} \; \frac{u^{-(\mathfrak{f}_{\mathfrak{p}}(\chi) + \mathfrak{f}_{\bar{\mathfrak{p}}}(\chi))} e_{\mathfrak{p}}(\chi) e_{\bar{\mathfrak{p}}}(\chi)}{u^{-\mathfrak{f}_p(\varrho)} \;\; e_p(\varrho)}$$

$$\cdot \; \frac{P_{\mathfrak{p}}(\bar{\chi}, u^{-1}) P_{\bar{\mathfrak{p}}}(\bar{\chi}, u^{-1}) P_{\mathfrak{p}}(\chi^c, u^{-1}) P_{\bar{\mathfrak{p}}}(\chi^c, u^{-1}) \qquad P_p(\varrho, w^{-1})}{P_p(\breve{\varrho}, u^{-1}) \qquad P_{\bar{\mathfrak{p}}}(\bar{\psi}\chi, \frac{1}{p}) \;\; P_{\bar{\mathfrak{p}}}(\bar{\psi}\chi^c, \frac{1}{p}) \; P_{\mathfrak{p}}(\bar{\psi}\chi^c, \frac{1}{p}) P_{\mathfrak{p}}(\bar{\psi}\chi, \frac{1}{p})}$$

$$\cdot \; \frac{L(\bar{\psi}\chi, 1) \cdot L(\bar{\psi}\chi^c, 1)}{L((\mathrm{Ind}\bar{\psi}) \otimes \mathrm{Ind}\chi, 1)} = \Omega_p^2 \; \alpha^2 \tau$$

with $\qquad \tau := \dfrac{\Omega_-}{\Omega_+}, \quad \alpha := \dfrac{\Omega_+}{\Omega} \in \mathcal{O}_{\mathfrak{p}}^{\times} \cap K.$

$$(\mathrm{Ind}\bar{\psi}) \otimes \mathrm{Ind}\chi = \mathrm{Ind}(\bar{\psi} \otimes \mathrm{Res}\ \mathrm{Ind}\chi) = \mathrm{Ind}(\bar{\psi} \otimes (\chi \oplus \chi^c))$$

$$u^{-1} = \frac{\psi(\mathfrak{p})}{p} = \frac{\bar{\psi}(\bar{\mathfrak{p}})}{p}, \quad w^{-1} = \frac{\psi(\bar{\mathfrak{p}})}{p} = \frac{\bar{\psi}(\mathfrak{p})}{p}$$

2. Case: $\dim(\varrho) = 1$

$$\frac{\mathcal{L}(\breve{\varrho})}{\mathcal{L}_E(\breve{\varrho})} = \ldots \frac{L(\bar{\psi}\varrho_{|G}, 1)}{L(E, \varrho, 1)} = \ldots \frac{L(\bar{\psi}\varrho_{|G}, 1)}{L(\mathrm{Ind}(\bar{\psi} \otimes \varrho_{|G}), 1)}$$

$$= \Omega_p \cdot \begin{cases} \alpha & \varrho(c) = +1 \\ & \text{if} \\ \alpha\tau & \varrho(c) = -1 \end{cases}$$

Here $\tau := \dfrac{\Omega_-}{\Omega_+}$, $\alpha := \dfrac{\Omega_+}{\Omega}$, $\alpha\tau = \dfrac{\Omega_-}{\Omega}$ all belong to $\mathcal{O}_{\mathfrak{p}}^{\times} \cap K$.

Need correction function

$$\mathcal{L}_{\mathrm{cor}}(\breve{\varrho}) = \frac{\mathcal{L}(\breve{\varrho})}{\mathcal{L}_E(\breve{\varrho})} = \begin{cases} \Omega_p^2\alpha^2\tau & & \varrho & = & \mathrm{Ind}\chi \\ \Omega_p\alpha & \epsilon\ D^{\times},\ \text{if} & \rho(c) & = & +1 \\ \Omega_p\alpha\tau & & \varrho(c) & = & -1 \end{cases}$$

for irreducible ϱ.

Indeed,

$$\mathcal{L}_{\mathrm{cor}} := \frac{\Omega_p\alpha}{2}\{(1+\tau) + (1-\tau)c\} \ \epsilon\ D[[\mathcal{G}]]^{\times}$$

works and the MC is proven!