Seminar: Geometric Structures on Manifolds

Dr. Gye-Seon Lee

Wintersemester 2015/2016

1. Time and Location

Tuesdays 2 - 4 p.m. (First meeting: 13 October 2015) and INF 288, HS 5

2. Description

About thirty years ago, the theory of 3-dimensional manifolds was revolutionized by work of Thurston. He showed that geometry has an important role to play in the theory in addition to the use of purely topological methods. This seminar aims at introducing students to some of these amazing developments and at discussing the various geometries which arise and their significance for the theory of 3-dimensional manifolds. At the organizational meeting I will propose several projects. During the first half of the semester, students will work on their project, which they will present during the second half of the semester. Every student is expected to hand in a short report as well.

3. Prerequisites

There are several topics for students to choose from, and hence any interested students are welcome to attend. The seminar will be taught in English.

4. Organization

Students who would like to sign up for the seminar should come to the organization meeting or send me an email (lee@mathi.uni-heidelberg.de) and let me know what project you would like to carry out. Once you have agreed on the topic of your project with me, you should contact me to discuss the content of your project in more detail. Please let me help you during the preparation of your presentation.

5. Schedule

The schedule of presentations will be updated on the web page:
http://www.mathi.uni-heidelberg.de/~lee/seminarWS1516.html

Tentative dates: (November 10); December (1), 8, 15, 22; January 12, 19, 26; (February 2)

6. Topics

(1) Hyperbolic geometry*
    [Thurston, Chapter 2] or [Benedetti and Petronio, Chapter A]
(2) Hyperbolic 2-manifolds
    [Benedetti and Petronio, Chapter B]
(3) Tessellations
    [Bonahon, Chapter 6]
(4) The Farey tessellation and circle packing*
    [Bonahon, Chapter 8]

(5) Kleinian groups*
[Bonahon, Chapter 10]
(6) The figure-eight knot complement*
[Bonahon, Chapter 11]
(7) Geometrization theorems in dimension 3*
[Bonahon, Chapter 12]
(8) Geometric structures on 2-dimensional orbifolds*
[Scott, §1 and §2]
(9) The basic theory of Seifert fibre spaces
[Scott, §3]
(10) The eight 3-dimensional geometries*
[Scott, §4]
(11) The classification of the 3-dimensional geometries*
[Scott, §5]
(12) Margulis' lemma
Reference: [Benedetti and Petronio, Chapter A]
(13) Hyperbolic Dehn surgery
Reference: [Ratcliffe, §10.5]
(14) Selberg's lemma
Reference: [Ratcliffe, §7.6]
(15) Geometric manifolds
[Thurston, Chapter 3]
(16) The structure of discrete groups
[Thurston, Chapter 4]

7. References

- Riccardo Benedetti and Carlo Petronio, *Lectures on hyperbolic geometry*, Springer (2003)
- Francis Bonahon, *Low-dimensional geometry : from Euclidean surfaces to hyperbolic knots*, American Mathematical Society (2009)
- John G. Ratcliffe, *Foundations of hyperbolic manifolds*, Second edition, Springer (2006)
- Peter Scott, *The geometries of* 3*-manifolds*, Bull. London Math. Soc. 15 (1983), 401–487
- William P. Thurston, *Three-dimensional geometry and topology*, Princeton University Press, 1997
- William P. Thurston*, *The geometry and topology of three-manifolds*, Electronic version, http://library.msri.org/nonmsri/gt3m/
- Marc Culler, Nathan M. Dunfield and Jeffrey R. Weeks, *SnapPy: a computer program for studying the topology of* 3*-manifolds*, http://snappy.computop.org