A restricted Magnus property for profinite surface groups (joint work with P. Zalesskii)

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Magnus proved that, given two elements x and y of a finitely generated free group F with equal normal closures $\langle x \rangle^F = \langle y \rangle^F$, then x is conjugated either to y or y^{-1}. More recently ([2] and [3]), this property, called the Magnus property, has been generalized to oriented surface groups.

In this paper, we consider an analogue property for profinite surface groups. While Magnus property, in general, does not hold in the profinite setting, it does hold in some restricted form. In particular, for $\mathscr{S}$ an extension closed variety of finite groups, i.e. closed by taking subgroups, homomorphic images and extensions, we prove that, if x and y are elements of the pro-$\mathscr{S}$ completion $\hat{\Pi}^{\mathscr{S}}$ of an orientable surface group Π, such that, for all $n \in \mathbb{N}$, it holds $\langle x^n \rangle^{\hat{\Pi}^{\mathscr{S}}} = \langle y^n \rangle^{\hat{\Pi}^{\mathscr{S}}}$, then x is conjugated to y^s for some $s \in (\hat{\mathbb{Z}}^{\mathscr{S}})^*$. As a matter of fact, a much more general property is proved and subsequently extended to a wider class of profinite completions.

The most important application of the above results is to provide a kind of linearization for the complex of profinite curves, introduced by the first author in [1]. This is applied to extend to profinite multi-twists the results of [1] on centralizers of profinite Dehn twists in the congruence completion of the Teichmüller group.

Another consequence is that, given a projective hyperbolic curve C, defined over a number field $\mathbb{k}$, the absolute Galois group of $\mathbb{k}$ acts faithfully on the direct limit of the first ℓ-adic cohomology groups of all Galois étale coverings of C.

References

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