Let F be a finite extension of Q_p. A mod p local Langlands correspondence is known for $GL_2(Q_p)$ and believed to exist between n-dimensional mod p representations of the absolute Galois group of F and certain mod p representations of $GL_n(F)$ for arbitrary F and n. A major obstacle to the study of the correspondence is that the supersingular mod p representations of $GL_n(F)$, which are the building blocks of the mod p representation theory of this group, are very poorly understood. After an introduction to the mod p representation theory of $GL_n(F)$, we will discuss recently discovered obstacles to the understanding of the supersingular representations, as well as what is known.