Let L/K be a finite Galois extension of number fields with Galois group G. If G is abelian, a classical conjecture of Brumer asserts that certain so-called Stickelberger elements (constructed via values of Artin L-functions) lie in the annihilator of the class group of L. We discuss how to generalize this conjecture to arbitrary G, and its relation to the equivariant Tamagawa number conjecture. Moreover, we present new resuls toward a proof of the (non-abelian) Brumer conjecture.