In arithmetic, Galois representations are one of the fundamental objects of interest and they arise quite naturally in several places.

The Galois representations coming from cuspidal automorphic forms on $\mathrm{GL}_n(\mathbb{A}_{\mathbb{Q}})$ are expected to be irreducible as representations of the absolute Galois group of Q. However,the local representations, obtained by restricting to a decomposition subgroup, can be reducible.

In this talk, we will show how a generalized notion of ordinariness for automorphic representations implies the reducibility of such local representations. We also show that non-ordinariness implies irreducibility in certain cases.

If time permits, we will also discuss the semisimplicity of local Galois representations attached to ordinary cuspidal eigenforms, following the approach of Ghate-Vatsal for odd primes, for $n = 2$ and $p = 2$.