In this talk, after introducing rationally connected varieties, I will discuss various ways in which a variety can be "connected by curves of a fixed genus". At least in characteristic zero, reduction to positive characteristic arguments prove that in the specific case of the existence of a single curve with large deformation space of morphisms to a variety implies that the variety is in fact rationally connected. Time permitting I will discuss attempts to show this result in positive characteristic.