HENGSTBERGER SYMPOSIUM

Higher Teichmüller theory and Higgs bundles: Interactions and new trends

2 – 6 November 2015

Contact:

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Monday, 2 November 2015

10:15 − 10:45        Coffee

10:45 − 11:00        Opening

11:00 − 12:00        Harmonic quasiisometric maps

Yves Benoist, University of Paris-Sud

We will see that any quasiisometric map between rank-one symmetric spaces lies within bounded distance of a unique harmonic map.   Joint work with D. Hulin.

12:00 − 14:00        Lunch

14:00 − 15:00        Degeneration of Hitchin representations

Tengren Zhang, Caltech

I will describe an analog of the Fenchel-Nielsen coordinates on the Hitchin component, and then use these coordinates to define a large family of deformations in the Hitchin component called "internal sequences". Then, I will explain some geometric properties of these internal sequences, which allows us to conclude some structural similarities and differences between the higher Hitchin components and Teichmüller space.

15:00 − 15:30        Coffee

15:30 − 16:30        Torelli theorems and integrable systems for parabolic Higgs bundle moduli spaces

Marina Logares, ICMAT

In the same way that the classical Torelli theorem determines a curve out of its polarised Jacobian, we will show that moduli spaces of parabolic bundles and parabolic Higgs bundles over a compact Riemann surface X also determine X. We will make use of a theorem of J. Hurtubise on the geometry of algebraic completely integrable systems in the course of the proof.   This is joint work with I. Biswas and T. Gómez.

17:00 − 19:30        Reception

Tuesday, 3 November 2015

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| 09:30 – 10:30 | SL(2,$\mathbb{C}$)-Higgs bundles with smooth spectral data |

Michael Wolf, Rice University

We show that for every nonelementary representation of a surface group into SL(2, $\mathbb{C}$) there is a Riemann surface structure such that the Higgs bundle associated to the representation lies outside the discriminant locus of the Hitchin fibration. Along the way in the argument, we encounter a number of constructions in the geometry of surfaces: complex projective structures, pleated surfaces, harmonic maps to $\mathbb{R}$-trees and the Thurston compactification. (Joint with Richard Wentworth.)

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| 10:30 – 11:00 | Coffee |

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| 11:00 – 12:00 | Convex projective structures on non-hyperbolic 3-manifolds |

Sam Ballas, UC Santa Barbara

A useful way to study hyperbolic structures on surfaces is to decompose the surface into pairs of pants, understand all the possible hyperbolic structures on a pair of pants, and then understand how such structures can be glued together. In higher dimensions, this technique no longer useful for studying hyperbolic structure. However in this talk we will see that these techniques can be generalized to study convex projective structures on non-hyperbolic 3-manifolds. This work is joint with Jeff Danciger and Gye-Seon Lee.

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| 12:00 – 14:00 | Lunch |

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| 14:00 – 15:00 | Milnor-Wood inequality for Higgs bundles and the Cayley correspondence |

Roberto Rubio, IMPA

In this talk we will show how the Jordan algebra structure arising in the tangent space of a non-compact Hermitian symmetric space G/H is responsible for the Milnor-Wood inequality for G-Higgs bundles over a compact Riemann surface. We will then look at maximal objects for G of tube type and present a rigidity result known as the Cayley correspondence: by choosing a $|Z(G)|$-root of the canonical bundle, the moduli space of polystable maximal G-Higgs bundles is isomorphic to the moduli space of polystable $K^2$-twisted H*-Higgs bundles, where H* is a non-compact dual of H. (Joint work with O. Biquard and O. Garcia-Prada.)

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| 15:00 – 15:30 | Coffee |

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| 15:30 – 16:30 | The geometry of symplectic Anosov representations |

Sara Maloni, Brown University

After revising the background theory of symplectic Anosov representations and their domains of discontinuity, we will focus on our joint work with D. Alessandrini and A. Wienhard. In particular, we will describe partial results about the homeomorphism of the quotient of the domain of discontinuity for quasi-Hitchin representations in Sp(4, $\mathbb{C}$) acting on the Lagrangian space Lag($\mathbb{C}^4$).

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| 19:00 – | Social Dinner |

Wednesday, 4 November 2015

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| 09:30 − 10:30 | Infinite framed flag configurations and integrable systems |

Vladimir Fock, University of Strasbourg

One of the way to introduce coordinates on moduli of framed SL(N) flat connections on a surface is to associate to a surface a configuration of infinitely many framed flags in an N-dimensional space equivariant with respect to the action of the fundamental group of the surface. The space of such configurations of the surface and then parameterize the latter by certain wedge products. The aim of this talk is to pass to configuration of flags in an infinite dimensional space and show that these configurations admit a similar description. Moreover if we impose invariance with respect to a larger discrete group we get a finite dimensional space. We will discuss possible geometric meaning of this space.

10:30 − 11:00    Coffee

11:00 − 12:00    Complex deformations of Anosov representations

Andrew Sanders, University of Illinois at Chicago

An Anosov representation of a hyperbolic surface group is a homomorphism from the surface group into a semi-simple Lie group which satisfies a certain dynamical property: from this property one deduces that Anosov representations are discrete, faithful and the set of all Anosov representations is an open subset of the space of all homomorphisms. In recent years, Guichard-Weinhard produced examples of co-compact domains of discontinuity for Anosov representations, which lie in various homogeneous spaces, thus giving an answer to the question of whether or not Anosov representations appear as monodromies of locally homogeneous geometric structures on manifolds. In this talk, which comprises joint work with David Dumas, I will discuss some of the complex analytic features of these locally homogeneous geometric manifolds in the case the relevant homogeneous space is a generalized flag variety. In particular, we will give sufficient conditions to compute the space of all infinitesimal deformations of the complex manifold underlying these manifolds. Time permitting, we will discuss the problem of deforming a pair (M, Z) where M is a holomorphic locally homogeneous manifold and Z is a complex sub-manifold and indicate an application to the study of Anosov representations.

Free afternoon

Thursday, 5 November 2015

09:30 − 10:30     Computing equivariant harmonic maps from $\mathbb{H}^2$ to $\mathbb{H}^3$

Brice Loustau, Rutgers University

I have been working with J. Gaster on a computer program which computes equivariant harmonic maps from $\mathbb{H}^2$ to $\mathbb{H}^3$. More precisely, given a closed surface group $\pi = \pi_1(S)$, a Fuchsian representation $\rho_0 : \pi \longrightarrow \mathrm{PSL}(2, \mathbb{R})$ and a reductive representation $\rho_1 : \pi \longrightarrow \mathrm{PSL}(2, \mathbb{C})$, the program computes and shows the unique $(\rho_0, \rho_1)$-equivariant harmonic map $f : \mathbb{H}^2 \longrightarrow \mathbb{H}^3$, using an *ad hoc* discrete flow method. The existence and uniqueness of this map is Donaldson-Corlette's theorem, and it is the key to the nonabelian Hodge correspondence between the character variety of the surface group and the moduli space of Higgs bundles. The broader goal of our project is to computationally investigate this correspondence.

10:30 − 11:00     Coffee

11:00 − 12:00     Higgs bundles and bounded cohomology

Andreas Ott , University of Heidelberg

Representations of fundamental groups of compact complex hyperbolic manifolds into Hermitian Lie groups have been studied using bounded group cohomology on the one hand and Higgs bundles on the other hand.   In this talk, we will discuss some aspects of the interplay between these two approaches, with a focus on Milnor-Wood type inequalities. This is joint work with Tobias Hartnick.

12:00 − 14:00     Lunch

14:00 − 15:00     Limits and degenerations of Higgs bundle moduli spaces

Jan Swoboda, University of Munich/Heidelberg

In the first part of this talk, I plan to report about recent joint work with Mazzeo, Weiß and Witt on the asymptotic behavior of solutions to Hitchin's self-duality equations in the limit of large Higgs fields. In a rather different direction, I shall focus in the second part on the behavior of the moduli space of solutions under degeneration of the underlying Riemann surface, which can be understood using gluing methods from geometric analysis.

15:00 − 15:30     Coffee

15:30 − 16:30     Hodge polynomials of $\mathrm{SL}(2, \mathbb{C})$ and $\mathrm{PGL}(2, \mathbb{C})$-character varieties

Javier Martínez, Complutense University of Madrid

The G-character variety of a surface of genus g is the moduli space parametrizing representations of the fundamental group of the surface into G. Twisted character varieties appear as natural generalizations of this concept: they are spaces of representations of the fundamental group of a punctured surface with fixed arbitrary holonomy around the puncture.  In the talk, we will show how to compute the E-polynomials of these spaces for arbitrary genus and G = $\mathrm{SL}(2, \mathbb{C})$, $\mathrm{PGL}(2, \mathbb{C})$, using a geometric technique based on stratifications and fibrations of the moduli space. (joint work with V. Muñoz)

Friday, 6 November 2015

$09:30 - 10:30$      Maximal SO(2,3) representations and beyond

Brian Collier, University of Illinois at Urbana-Champaign

For a closed surface S of genus $g > 1$, the space of maximal $PSp(4,\mathbb{R})$ representations is especially diverse. For example, there are $2(2^{2g} - 1) + 4g - 3$ connected components, and for each integer $0 < d < 4g - 3$ there is a particularly interesting smooth connected component of the character variety which we call a Gothen component. When $d = 4g - 4$ the Gothen component is the Hitchin component and when $d < 4g - 4$ the Gothen components are noncontractible and contain only Zariski dense representations. Generalizing Labourie's results for Hitchin representations, we will give a mapping class group invariant parameterization of the Gothen components as fiber bundles over Teichmüller space. For $n > 2$ there is no component of the maximal $PSp(2n, \mathbb{R})$ representations which generalize the Gothen representations. However, motivated by the isomorphism $PSp(4, \mathbb{R})=SO(2,3)$, we will use a Higgs bundle description of the Gothen components to show that the Gothen representations are an SO(n,n+1) phenomenon.

$10:30 - 11:00$      Coffee

$11:00 - 12:00$      Cluster theory for non-orientable surfaces

Frederic Palesi, Aix-Marseille University

In their seminal work, Fock and Goncharov defined coordinates on the spaces of framed and decorated representations of an orientable surface with boundaries into $PGL(m, \mathbb{C})$. They discovered that the appropriate combinatorial framework for this is closely related to cluster algebras, and that various structures (Poisson and symplectic structure, positivity, etc ...) are easily described in this context. The goal of this talk will be to give a brief introduction to the formalism of cluster algebra and explain the relations with higher Teichmüller theory. Then we will explain how we can generalize this to the non-orientable case.