RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

Mathematisches Institut

Vorlesung Geometrische Gruppentheorie
Heidelberg, 15.11.2013

Exercise sheet 5
Quasi-Isometries
To hand in by Friday November 22, 13:00

Exercise 1. Give a detailed proof that the metric spaces $\mathbb{R}$ and $[0, \infty)$ are not quasi-isometric. (The subset $[0, \infty) \subset \mathbb{R}$ inherits the metric from $\mathbb{R}$).

Exercise 2. Prove that the metric spaces $\mathbb{R}^m$ and $\mathbb{R}^n$ are not quasi-isometric if $n \neq m$. (You can use, without the need of proving it, the Borsuk-Ulam theorem, that says that every continuous map from $\mathbb{S}^n$ to $\mathbb{R}^n$ maps some pair of antipodal points to the same point).

Exercise 3. Let T_m be a tree such that every vertex has m incident edges, with the metric that gives length 1 to every edge. Prove that the trees T_m and T_n, with $m \neq n$ are quasi-isometric. Are they bilipschitz equivalent?

Exercise 4. Let $G = (V, E)$ be a finite connected graph with $V \neq \emptyset$. Show that G is a tree if and only if

$$\#(E) = \#(V) - 1$$

Exercise 5. Are the metric spaces $\mathbb{Z}$ and $\{n^3 \mid n \in \mathbb{Z}\}$ quasi-isometric? (The subset $\{n^3\} \subset \mathbb{Z}$ inherits the metric from $\mathbb{Z}$).

Exercise 6 (Bilipschitz equivalences and quasi-isometries)**.**

1. Show that a bijective quasi-isometry between finitely generated groups with respect to the word metric is a bilipschitz equivalence.
2. Are all bijective quasi-isometries between general metric spaces bilipschitz equivalences?